THE SCIENCE OF MUSICAL SOUNDS

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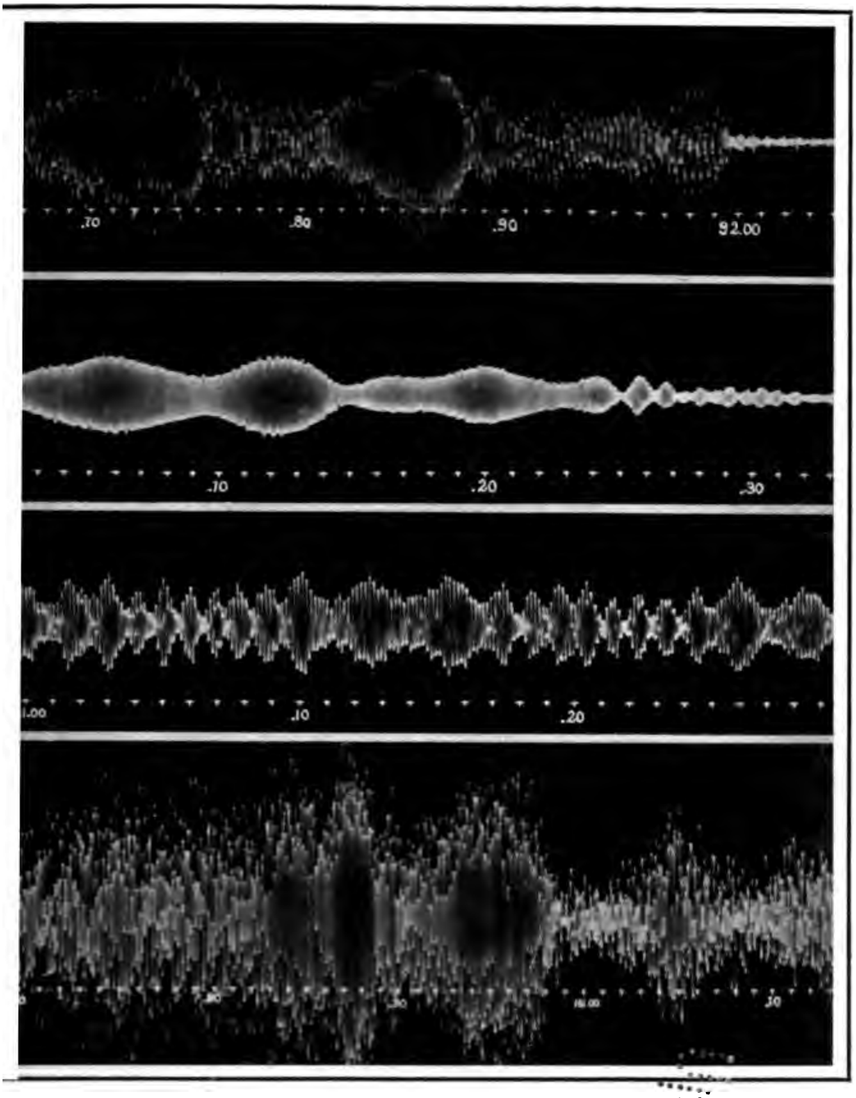

Lucia di Lammermoor" as sung score. The dots are time signals e of the second line is from the s nearly four times as long as the mplex wave due to the six solo

THE SCIENCE OF MUSICAL SOUNDS

BY

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New York
THE MACMILLAN COMPANY
1916

Set up and printed. Published March, 1916.

Norwood Press
J. S. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

PREFACE

A SERIES of eight lectures was given at the Lowell Institute in January and February, 1914, under the general title of "Sound Analysis." These lectures have been rewritten for presentation in book form, and "The Science of Musical Sounds" has been chosen for the title as giving a better idea of the contents. They appear substantially as delivered, though some slight additions have been made and much explanatory detail regarding the experiments and illustrations has been omitted. The important additions relate to the tuning fork in Lecture II and harmonic analysis in Lecture IV, while two quotations are added in the concluding section of Lecture VIII.

A course of scientific lectures designed for the general public must necessarily consist in large part of elementary and well known material, selected and arranged to develop the principal line of thought. It is expected that lectures under the auspices of the Lowell Institute, however elementary their foundation, will present the most recent progress of the science. The explanations of general principles and the accounts of recent researches must be brief and often incomplete; nevertheless it is hoped that the lectures in book form will furnish a useful basis for more extended study, and to further this end they are supplemented by references to sources of additional information. The references are collected in an appendix, citations being made by numbers in the text corresponding to the numbers in the appendix.

It is further expected that such lectures will be accompanied by experiments and illustrations to the greatest possible degree; the nature and extent of this illustrative material is shown as well as may be by the aid of diagrams and pictures, nearly all of which have been especially prepared, and much care has been taken to make them as expressive as possible of the original demonstrations and explanations.

The methods and instruments used in sound analysis by the author, and many of the results of such work, were described in the lectures in advance of other publication and it is the intention to supplement the brief accounts here given by more detailed reports in scientific journals.

The author is greatly indebted to many friends for the kindly interest shown during the progress of the experimental work here described; and he is especially under obligation to Professor Frank P. Whitman of Western Reserve University, and to Mr. Eckstein Case and Professor John M. Telleen of Case School of Applied Science, for many helpful suggestions received while the manuscript was in preparation.

DAYTON C. MILLER.

CLEVELAND, OHIO,
July, 1915.

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THE SCIENCE OF MUSICAL SOUNDS

LECTURE I

SOUND WAVES, SIMPLE HARMONIC MOTION, NOISE AND TONE

INTRODUCTION

WE are beings with several senses through which we come into direct relation with the world outside of ourselves. Through two of these, sight and hearing, we are able to receive impressions from a distance and through these only do the fine arts appeal to us; through sight we receive the arts of painting, sculpture, and architecture, and through hearing, the arts of poetry and music.

Undoubtedly music gives greater pleasure to more people than does any other art, and probably this enjoyment is of a more subtle and pervading nature; every one enjoys music in some degree, and many enjoy it supremely. Sound is also of the greatest practical importance; we rely upon it continually for the protection of our lives, and through talking, which is but making sounds according to formula, we receive information and entertainment. These facts give ample justification for studying the nature of sound, the material out of which music and speech are made.

The study of sounds in language is as old as the human race, and the art of music is older than tradition, but the science of music is quite as modern as the other so-called modern sciences. Sound being comparatively a tangible phenomenon, and so intimately associated with the very existence of every human being, one would expect that if there are any unknown facts relating to it, a large number of investigators would be at work trying to discover them. There has been in the past, as there is now, a small number of enthusiastic workers in the field of acoustics who have accomplished much; but it is no doubt true that this science has received less attention than it deserves, and especially may this be said of the relation of acoustics to music.

Sound Defined

Sound may be defined as the sensation resulting from the action of an external stimulus on the sensitive nerve apparatus of the ear; it is a species of reaction to this external stimulus, excitable only through the ear, and distinct from any other sensation. Atmospheric vibration is the normal and usual means of excitement for the ear; this vibration originates in a source called the *sounding body,* which is itself always in vibration.

The source may be constructed especially to produce sound; in a stringed instrument the string is plucked or bowed and its vibration is transferred to the soundboard, and this in turn impresses the motion upon a larger mass of air; in the flute and other wind instruments the air is set in motion directly by the breath. The vibration often originates in bodies not designed for producing sounds, as is illustrated by the squeak and rumble of machinery.

The physicist uses the word *sound* to designate the vibrations of the sounding body itself, or those which are set up by the sounding body in the air or other medium and which are capable of directly affecting the ear even though there is no ear to hear.

THE NATURE OF SOUND

There are numerous experiments which demonstrate that a sounding body vibrates vigorously. When a tuning fork, Fig. 1, is struck with a soft felt hammer, it gives forth a continuous sound. The fork vibrates transversely several hundred times a second, though the distance through which it moves is only a few thousandths of an inch. These movements, which are too minute and rapid to be appreciated by the eye, may be made evident by means of a pith-ball pendulum adjusted to rest lightly against the prong; when the fork is sounding, the ball is violently thrown aside.

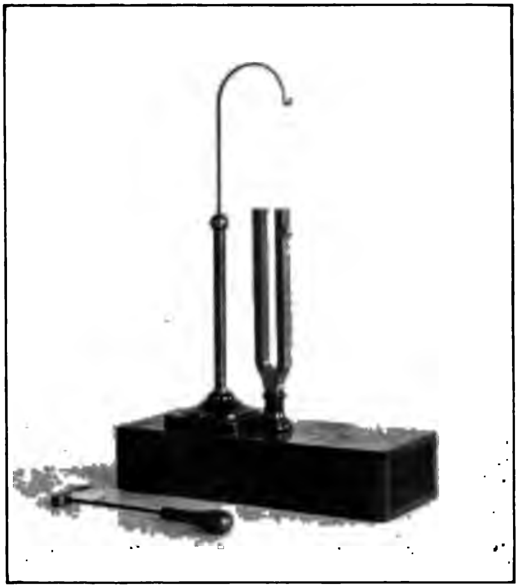


Fig. 1. Tuning fork and pith ball for demonstration of vibration.

The powerful longitudinal vibrations of a metal rod may be exhibited in a like manner with apparatus arranged as shown in Fig. 2. If a piece of rosined leather is drawn along the rod a loud tone is emitted, and the ivory ball resting lightly against the end is thrown high in the air. At the center of the rod the molecules are at rest, forming a node, while at the ends the vibrations are of relatively large amplitude. The molecules vibrate hundreds of times per second, and the comparatively feeble movements of single molecules, by their cumulative effects in the mass, develop enormous forces, equivalent to a tensile force of several tons. If a glass tube, held at its middle, is rubbed with a piece of wet cloth, the longitudinal vibrations are often so vigorous as to cause the tube to separate into many pieces.

A glass bell may be set into transverse vibration by bow-

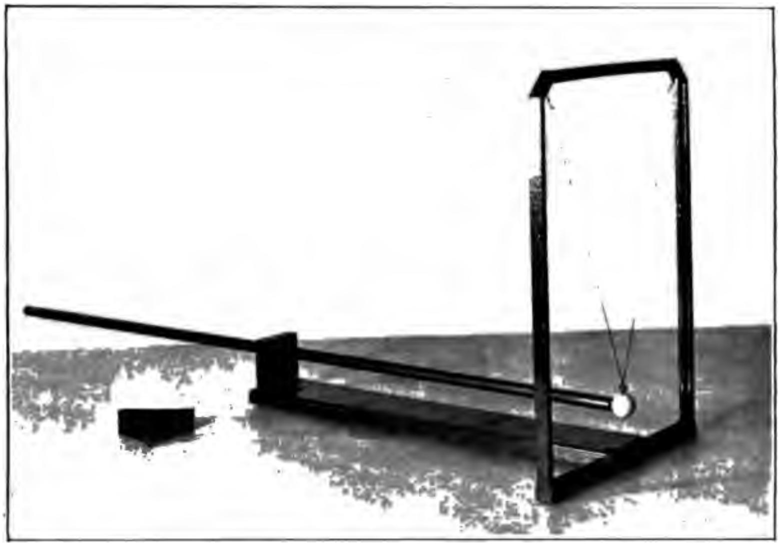


FIG. 2. Longitudinally vibrating rod.

ing across the edge. The vibrations cause periodic deformations of the shape, from a circle to an ellipse and back to the circle. The circumference must vibrate in at least four segments, with the formation of loops and nodes. Balls, suspended from a revolving support, as shown in Fig. 3, may rest against the surface of the bell and may be used to locate the loops and nodes. The vibration may become so violent as to shatter the glass.

The vibrations of the source produce various physical effects in the surrounding air, such as displacements, velocities, and accelerations, and changes of density, pressure, and temperature; because of the elasticity of the air, these displacements and other phenomena occur periodically and are transmitted from particle to particle in such a manner

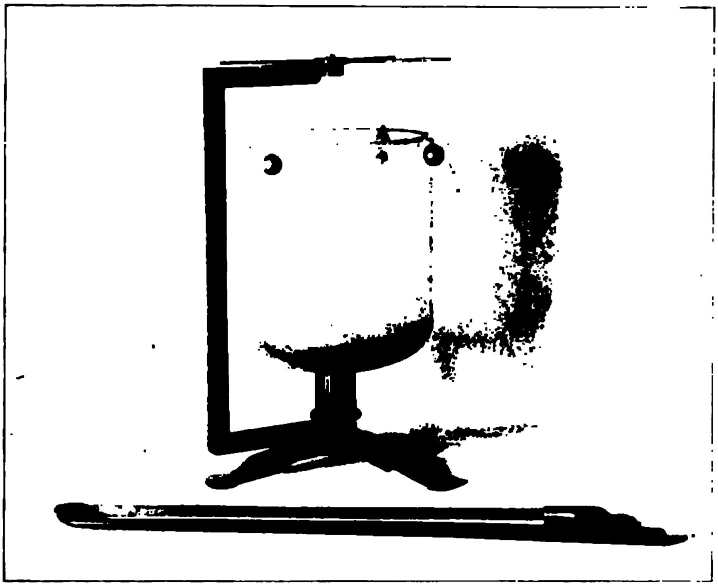


FIG. 3. Glass bell.

that the effects are propagated outward from the source in radial directions. These disturbances of all kinds, as they exist in the air around a sounding body, constitute ***sound waves.*** The velocity of sound is about 1132 feet per second when the temperature of the air is 70° F., a temperature common in auditoriums; at the freezing temperature, 32° F., it is about 1090 feet per second. Musical sounds of different pitches are all propagated in the open air with the same velocity. Explosive sounds and sounds confined, as in tubes, are propagated with different velocities.[1] Wave disturbances may be transmitted by solid and liquid as well as gaseous matter, but our present study relates mainly to what may be heard, and the explanations are limited for the most part to certain features of waves in air, and particularly to the nature of the movements of the air particles when transmitting musical sounds.

Simple Harmonic Motion and Curve

The simplest possible type of vibration which a particle of elastic matter of any kind may have is called *simple harmonic motion;* it takes place in a straight line, the middle of which is the position of rest of the particle; when the particle is displaced from this position, elasticity develops a force tending to restore it, which force is *directly proportional to the amount of the displacement;* if the displaced particle is now freely released, it will vibrate to and fro with simple harmonic motion. The name originated in the fact that musical sounds in general are produced by complex vibrations which can be resolved into component motions of this type.

Other forces than those of elasticity may act in the manner described, as for instance the action of the force of gravity on the bob of a pendulum; if the bob is considered as swinging in a straight line, it has simple harmonic motion, which is also called *pendular motion.*

Simple harmonic motion has several evident features: it takes place in a straight line; it is vibratory, moving to and fro; it is periodic, repeating its movements regularly; there are instants of rest at the two extremes of the movement; starting from rest at one extreme the movement quickens till it reaches its central point, after which it slackens in reverse order, till it comes to rest at the other extreme. The speed of the particle so moving, the rate at which the speed changes, and other features are very important in a complete study of simple harmonic motion, but for our purpose we need give only a few simple definitions.

The *frequency* of a simple harmonic motion is the number of complete vibrations to *and* fro per second; the *period* is the time required for one complete vibration; the *amplitude* is the range on one side or the other from the middle point of the motion, therefore it is half the extreme range of vibration; the *phase* at any instant is the fraction of a period which has elapsed since the point last passed through its middle position in the direction chosen as positive.

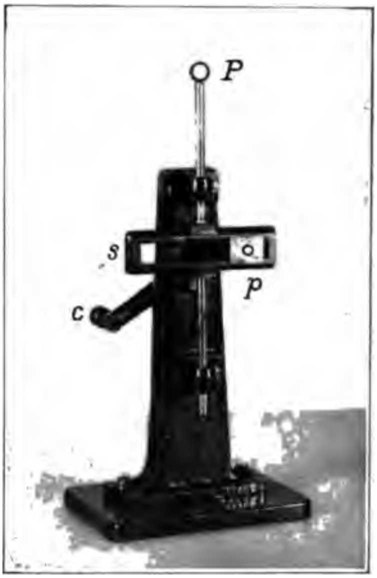


FIG. 4. Simple harmonic motion from mechanical movement.

Simple harmonic motion is approximated in various mechanical movements, while a few simple machines reproduce it exactly;[2] this reproduction is always accomplished by a transformation of uniform motion in a circle into rectilinear motion. The pin-and-slot device has a slotted frame *s*, Fig. 4, which is movable up and down only; the pin *p* of the crank *c* moves in the slot; when the crank is turned with uniform angular speed, the frame and all rigidly attached parts, such as the point P, move with simple harmonic motion. The usual starting point for this motion is the middle position of P when it is about to move upward, that is, when the crank is horizontal with the pin at the extreme right and about to turn counterclockwise. One complete vibration is produced when the crank makes one revolution and the point P moves from its mid-position to the extreme upper position, down to the lower extreme, and back to mid-position. The *period* is the time required for the complete vibration, that is, for one revolution of the crank; the *phase* at any instant is the fraction of a period which has elapsed since the point last passed through the starting point, and is often expressed by the number of degrees through which the crank has turned in the interval, as is further illustrated in Fig. 6; the *amplitude* is measured by half the extreme movement, that is, by the length of the crank from center to pin. This device is used in several of the harmonic synthesizers described in Lecture IV.

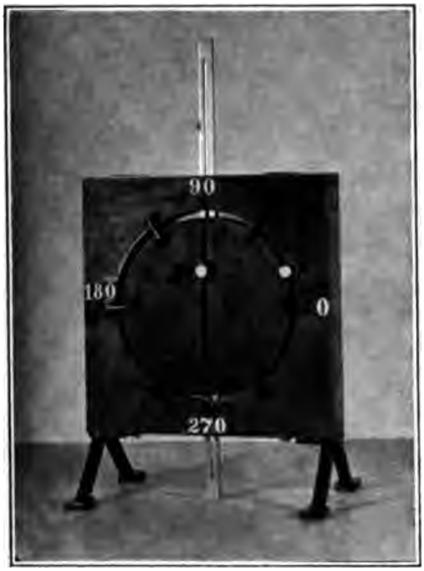


FIG. 5. Relation of simple harmonic and circular motion.

In treatises on mechanics simple harmonic motion is often defined as the projection of uniform motion in a circle upon a diameter of the circle; this definition is illustrated by the form of the pin-and-slot apparatus shown in Fig. 5. Turning the crank on the back of the apparatus causes the point *P* in the diameter to move up and down with a true harmonic

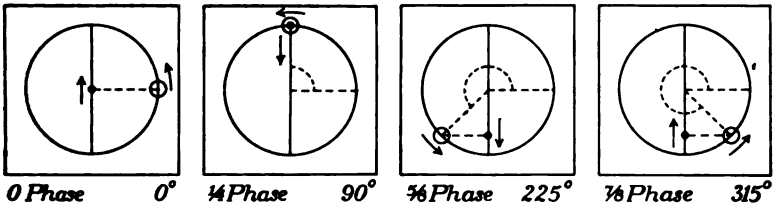


FIG. 6. Phases of simple harmonic motion.

motion when the point *p* in the circle revolves with uniform speed; the two points are always in the same horizontal line, or the point in the diameter is always the projection of the one in the circle; Fig. 6 illustrates the motion in various phases.

A crank pin *p* is pivoted in the center of a rod *AB*, Fig. 7; the ends of the rod are pivoted to sliders which move in two perpendicular, straight grooves; when the crank is turned with uniform speed, both of the points *A* and *B* move with simple harmonic motion. The displacement of either slider from its central position is always twice the displacement of the projection of the point *p* on the corresponding groove.

FIG. 7. Simple harmonic motion from mechanical movement.

A simple harmonic motion can be obtained without the friction of sliders in grooves by employing a pantograph to give the arithmetical mean of two equal and opposite circular motions, as suggested by Everett.[2] When the crank *c*, Fig. 8, is turned, the point *P* moves up and down in a straight line, so that it is always in the horizontal line con-

FIG. 8. Simple harmonic motion from mechanical movement.

FIG. 9. Simple harmonic motion from mechanical movement.

necting the points *A* and *B*, and therefore, when the wheels rotate with uniform speed, it has simple harmonic motion.

A simple harmonic motion is given to any point *P* on the circumference of a wheel, Fig. 9, when the wheel rolls with uniform speed on the inside of an annulus *a*, the radius of which is equal to the diameter of the wheel. The point *P* is always in the horizontal line passing through the point of contact *a* of the wheel and annulus.

SIMPLE HARMONIC MOTION

The movement of a sliding block connected to a crank by a pitman rod, as the crosshead of an engine, has a distorted simple harmonic motion, the errors of which may be corrected by suitable mechanism; Fig. 10 shows a device due to Smedley,[3] having two crossheads, c_1 and c_2, on opposite sides of the crank pin; during the motion these are oppositely displaced from the true harmonic positions, and the errors are equalized by a system of levers acting on the central

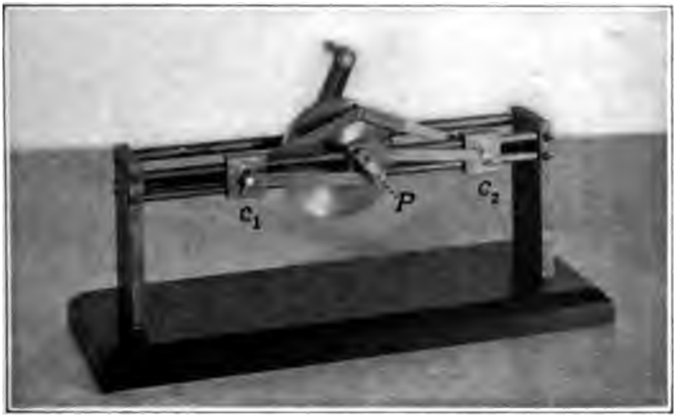


FIG. 10. Simple harmonic motion from compensated crosshead movement.

sliding block P, which receives simple harmonic motion when the crank revolves uniformly.

A simple harmonic motion combined with a uniform motion of translation traces a *simple harmonic curve;* this condition is illustrated by a pendulum swinging from a fixed point, Fig. 11, and leaving a trace on a sheet of paper moving underneath. The simple harmonic curve, Fig. 12, is perfectly simple, regular, and symmetrical; in mathematical study it is frequently referred to as a *sine curve;* a curve of the same form but differing in phase by a quarter period, or 90°, is a *cosine curve.* As explained in the next section, such a curve is an instantaneous representation of the condition of motion in a simple wave. Various terms used with regard to simple harmonic motion are also applicable to the curve; the amplitude is the height of a crest above the axis, Fig. 12; the period is the time required to trace one wave length consisting of a crest and trough; the frequency is the number of periods, or wave lengths traced, per second; the phase varies along the axis, passing through a complete cycle in one wave length;

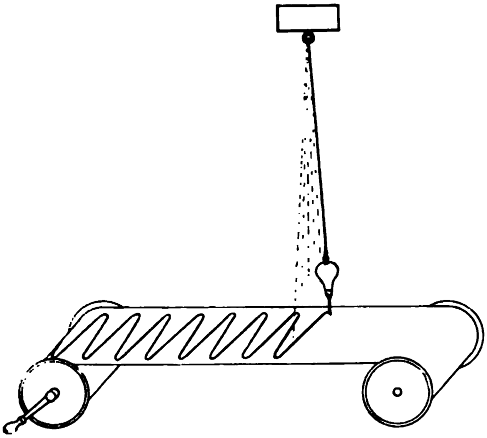


FIG. 11. Tracing a simple harmonic curve.

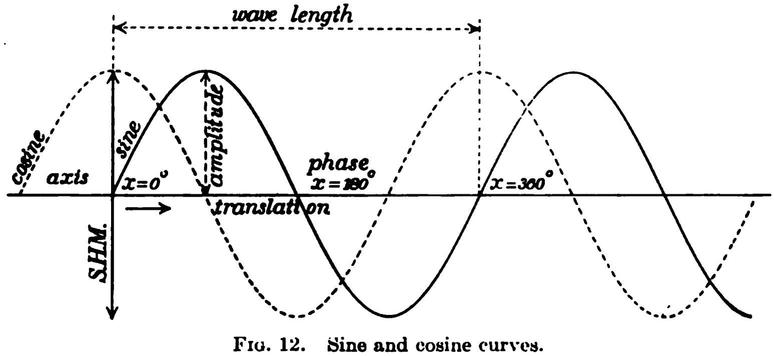


FIG. 12. Sine and cosine curves.

the velocity is the rate of translation and is equal to the wave length multiplied by the number of waves per second.

Sine curves may differ considerably in appearance, depending upon the relation of amplitude and wave length (frequency), though all must have the same general properties and be equally regular and simple. All the curves shown in Fig. 13 are simple harmonic, or sine curves, and differ only in amplitude A and frequency n, the relative values of these quantities being shown in the figure.

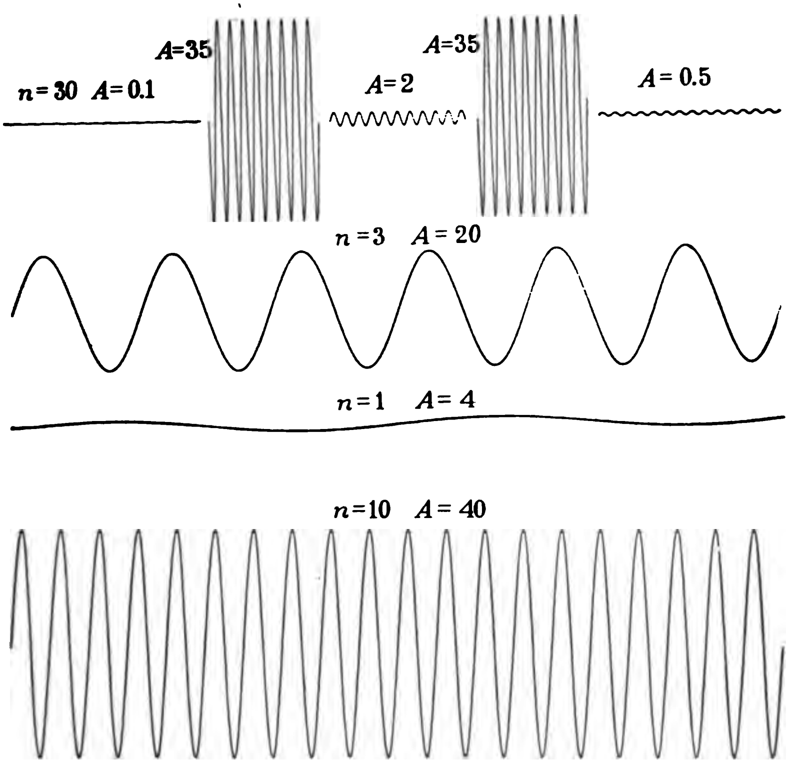


FIG. 13. Sine curves of various dimensions.

WAVE MOTION

The essential characteristic of *wave motion* is the continuous passing onward from point to point in an elastic medium of a periodic vibration which is maintained at the source. These vibrations, being periodic, produce a series of waves following each other at regular intervals, the speed of propagation depending upon the elastic properties of the medium. There are two distinct motions involved: the vibration of the individual particles about their positions of rest and the progressive outward movement of the wave form. The source of a wave motion may be a disturbance

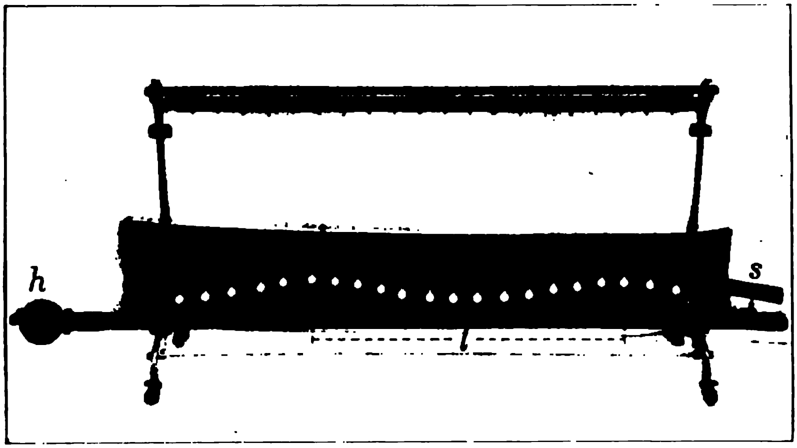


FIG. 14. Machine for illustrating transverse waves.

of any type, but for sound waves it consists of vibratory movements which are either simple harmonic or compounds of such.

A simple *transverse wave motion* is represented by the wave machine shown in Fig. 14; the successive pendulums are given similar periodic transverse vibrations in successive times by a slider *s*, which is moved from left to right by turning the handle *h*. The slider produces a wave crest which moves along the row of balls and disappears, being followed periodically by other crests; the velocity of wave propagation is the velocity with which the slider is moved; the wave length is the actual distance l from crest to crest of the wave; amplitude, period, and frequency are illustrated in the vibrations of the pendulums.

One of the bars to which is attached one string of each bifilar suspension of a pendulum may be shifted lengthwise and away from the other bar so that the pendulums can vibrate only in a longitudinal direction; by moving a slider

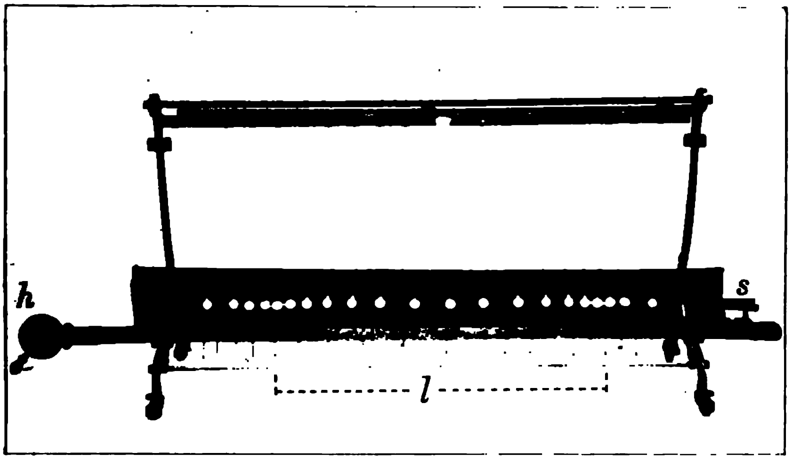


Fig. 15. Machine for illustrating longitudinal waves.

of a second form s, Fig. 15, simple harmonic motions, exactly the same as before except that they are in the direction of propagation, are given to the series of pendulums; this produces, instead of the crests and troughs of the former wave, condensations and rarefactions in the spacing of the particles which follow each other periodically, and moving forward with the velocity of wave propagation illustrate a *longitudinal wave motion*. In this case the wave length is the distance from one condensation to the next, and the various other characteristics are substantially the same as for the transverse wave. In fact, one type of wave can be transformed into the other and back again by merely shifting one of the suspending bars while the pendulums are vibrating; this turns the direction of vibration of each pendulum without disturbing the character of the motion.

The simple harmonic curve may be considered an instantaneous representation of a transverse wave; it shows by its shape the nature of the periodic vibration and exhibits the displacements and conditions of motion of a continuous series of particles transmitting the wave. In Fig. 16, *A*

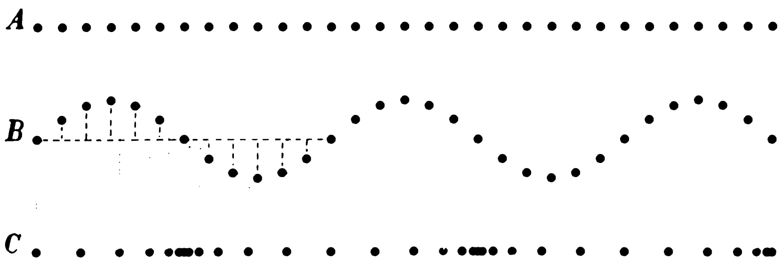


FIG. 16. Transverse and longitudinal displacements.

represents a row of particles at rest; if a transverse wave is being transmitted, the particles at some instant will be displaced as shown in *B*, forming a harmonic curve. If the displacements are of the same amounts but occur in a longitudinal direction, upward displacements in *B* corresponding to forward displacements in *C*, and vice versa, there results a longitudinal wave of condensation and rarefaction, or of pressure changes; this is the type of sound waves in air. Sound waves usually pass outward from the source in the form of expanding surfaces of disturbance, and the nature of the pressure changes, as applied to surfaces, may be illustrated by the spacing of the lines in *B*, Fig. 17. The relations of condensation and rarefaction of the longitudinal wave to crest and trough of the transverse wave are shown in both Figs. 16 and 17.

The amounts of displacement, the amplitudes, periods, frequencies, and velocities of propagation are defined in exactly the same way in the two types of waves; only the directions of displacement differ. It can be shown that both kinds of wave motion are adequately and correctly represented by the harmonic curve. The curve B, Fig. 16, conveys to the eye a much clearer idea of the displacements than does C, though

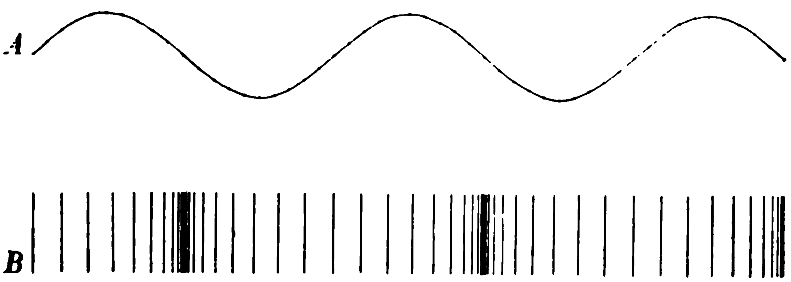


FIG. 17.—Wave of compression.

the displacements of the successive particles are of exactly the same amount in both instances. Nearly all of the waves to be studied in these lectures are of the longitudinal type, but they will be represented by curves of transverse displacement.

As will be more fully developed in later lectures, several simple harmonic motions of various amplitudes, frequencies, and phases, moving in the same or different directions, often coexist, producing wave motions which are represented by curves of very complex shapes.

Sound waves in solids may be either transverse or longitudinal, but the properties of liquids and gases are such that only longitudinal displacements, or pressure changes, can be

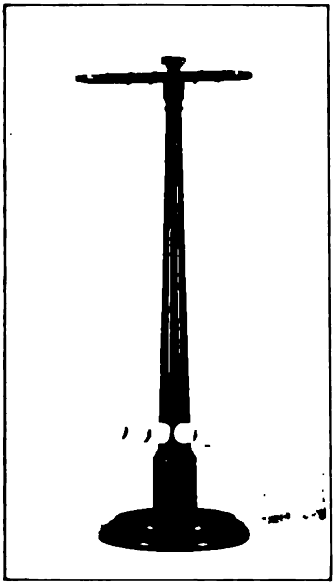


Fig. 18. Collision balls.

transmitted as wave motions. The transmission of a longitudinal wave through a solid or liquid body is illustrated by the collision balls, Fig. 18. When the medium is very compressible, such as a gas, the method of propagation is better shown by the apparatus, Fig. 19, which consists essentially of a long, flexible spring suspended so as to move horizontally; if a push of compression is given to one end of the spring, it will be transmitted as a wave in a manner to be easily followed by the eye.

An illustration of two simple harmonic motions at right angles is given by the compound pendulum apparatus shown in Fig. 20, the bob of which is a weight carrying a glass vessel

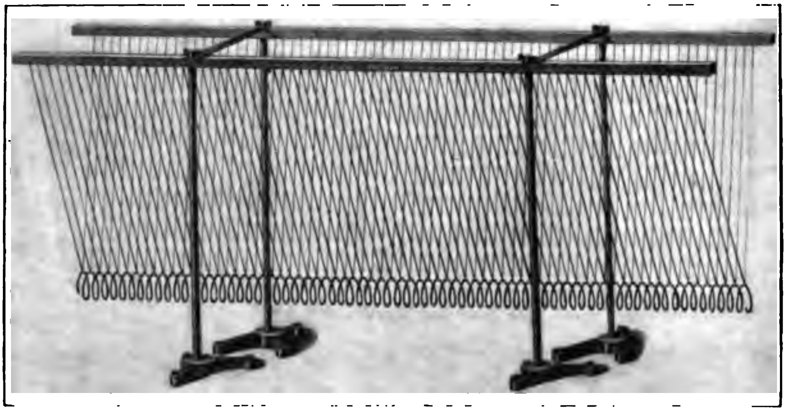


Fig. 19. Apparatus for illustrating a wave of compression.

containing sand; as the pendulum swings the sand flows from a small aperture in the bottom of the vessel and leaves a trace on the paper underneath. The pendulum may swing to and fro and from side to side; for the first movement its length is l_1, but on account of the arrangement of the two suspending strings, its length for sidewise movement is l_2; hence the periods of the two movements are different and the bob swings in a peculiar curve compounded of two simple move-

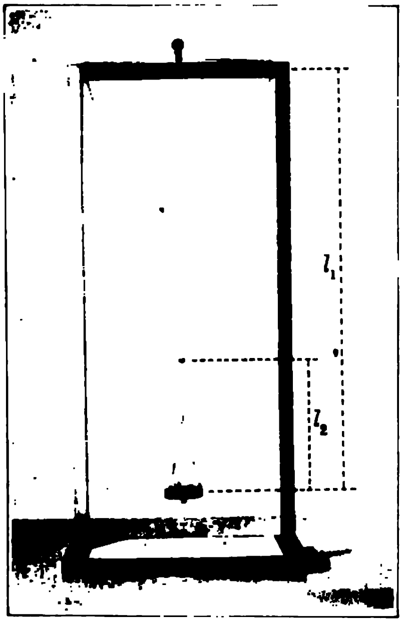


FIG. 20. Compound pendulum.

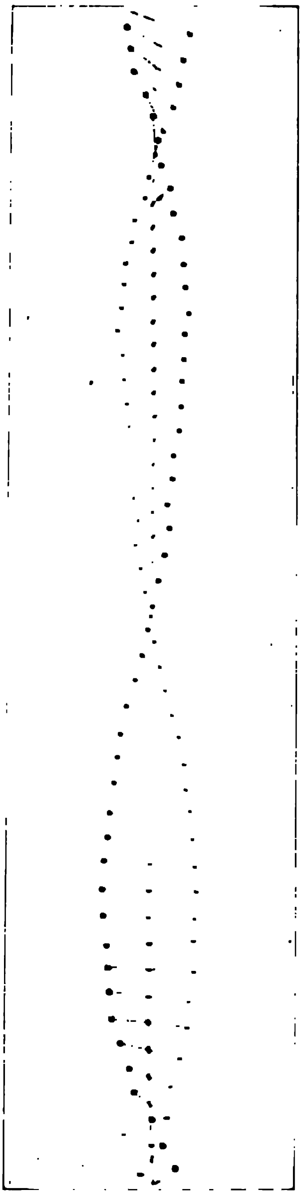


FIG. 21. Torsional wave.

ments; the curve shown in the figure results from periods of the exact ratio of 2 : 3; other ratios give characteristic figures; these curves are known as *Lissajous's figures.*[14] Compound harmonic motion of this kind is made use of in accurate tuning, as described in Lecture II.

Various other types of motion besides simple harmonic may generate waves; a *torsional wave,* consisting of angular harmonic motion, may be transmitted by a loaded wire, as shown in Fig. 21.

The Ear

Sound has been defined as the sensation received through the ear, and the definition has been extended to include the external cause of the sensation. All that the ear perceives in the complex music of a grand opera or of a symphony orchestra is contained in the wave motion of the air consisting of periodic changes in pressure and completely represented by motion of one dimension, that is, by motion confined to a straight line; or, as Lord Kelvin has expressed it, sound is "a function of one variable."

That motion of one dimension is capable of producing these sounds is amply proved by the talking machine; in the cylinder type of machine the tracing point moves up and down, and gives a backward-and-forward motion to the diaphragm, each point of which moves in a straight line; the resulting wave of compression is transmitted by the air to the eardrum. The telephone is another demonstration of the same fact. Some of the disk types of talking machines not only illustrate the movement in one direction, but also demonstrate that a transverse vibration on the record is transformed, through the needle and connecting levers, into an equivalent longitudinal motion at the diaphragm of the sound box. It is marvelous that complex musical results can be produced by such seemingly simple mechanical means.

The functioning of the ear, which is a wonderfully complex organ, Fig. 22, is but imperfectly understood; physiologists are studying its structure, and psychologists are investigating the manner of the reception and perception of the sensation of sound; the study of these most interesting questions is quite outside of the province of these lectures, which is confined to the physics of that which may be heard.

FIG. 22. Model of the ear, dissected.

that is, to sound waves as they exist in the air and to their sources.

The ear divides sounds roughly into two classes: *noises*, which are disagreeable or irritating, and *tones*, which are received with pleasure or indifference.

NOISE AND TONE

Noise and *tone* are merely terms of contrast, in extreme cases clearly distinct, but in other instances blending; the difference between noise and tone is one of degree. A simple tone is absolutely simple mechanically; a musical tone is more or less complex, but the relations of the component tones, and of one musical sound to another, are appreciated by the ear; noise is a sound of too short duration or too complex in structure to be analyzed or understood by the ear.

The distinction sometimes made, that noise is due to a non-periodic vibration while tone is periodic, is not sufficient; analysis clearly shows that many so-called musical tones are non-periodic in the sense of the definition, and it is equally certain that noises are as periodic as are some tones. In some instances noises are due to a changing period, producing the effect of non-periodicity; but by far the greater number of noises which are continuous are merely complex and only apparently irregular, their analysis being more or less difficult.

The ear, because of lack of training or from the absence of suitable standards for comparison or perhaps on account of fatigue, often fails to appreciate the character of sounds and, relaxing the attention, classifies them as noises.

Small sticks of resonant wood may be prepared, Fig. 23, such that when dropped, the resulting sound is a mixture of noise and simple musical tones. If several of these sticks are dropped together, the sound gives the effect of noise only, while if the sticks are dropped one at a time in proper order, the ear clearly distinguishes a musical melody in spite of the accompanying noise. The drawing of a cork from a bottle expands the contained air; when the cork is wholly withdrawn, the air, because of its elasticity, vibrates with a frequency dependent upon the size and shape of the bottle. The resulting sound is of short duration and is thought of only as a popping sound, while it is in reality a musical tone. The musical characteristic is made evident by drawing the plugs from several cylindrical bottles, Fig.

23, the tones of which are in the relations of the common chord, *do*, *mi*, *sol*, *do*. A distinguishable tune can be played on a flute without blowing into it, the air in the tube being set in vibration by snapping the keys sharply against the proper holes to give the tune.

A conspicuous instance of the change in classification of a musical composition from noise to music is provided by Wagner's "Tannhäuser Overture." After this overture had been known to the musical public for ten years it was criti-

FIG. 23. Sticks and bottles which produce musical noises.

cized in the *London Times* as "at best but a commonplace display of noise and extravagance." A Frankfort (Germany) critic said in 1853 that "'Tannhäuser,' so far as the public is concerned, may be considered a thing of the past." It was called "shrill noise and broken crockery effects." The eminent musical pedagogue, Moritz Hauptmann (1846), pronounced it "quite atrocious, incredibly awkward in construction, long and tedious. It seems to me," he says, "that a man who will not only write such a thing, but actually have it engraved, has little call for an artistic career."

Sidney Lanier, the poet-musician, who better understood this composition, wrote in a letter to his wife,[4] "Ah, how they have belied Wagner! I heard Thomas' orchestra play his overture to 'Tannhäuser.' The 'Music of the Future' is surely thy music and my music. Each harmony was a chorus of pure aspirations. The sequences flowed along, one after another, as if all the great and noble deeds of time had formed a procession and marched in review before one's *ears*, instead of one's *eyes*. These 'great and noble deeds' were not deeds of war and statesmanship, but majestic victories of inner struggles of a man. This unbroken march of beautiful-bodied Triumphs irresistibly invites the soul of man to create other processions like it. I would I might lead so magnificent a file of glories into heaven!"

As compared with the usual composition of its time "Tannhäuser Overture" must be considered as having a complicated construction. There is an accompaniment, quite independent of the main theme, which forms a beautiful background of tone, upon which the noble melody is projected. Many of the early listeners may have given their attention to this accompaniment and so have lost the impressiveness of the melody; to them it was a confused mass of tone producing the effect of noise.

The study of noises is essential to the understanding of the qualities of musical instruments, and especially of speech. Words are multiple tones of great complexity, blended and flowing, mixed with essential noises. If with the vowel tone *ă* (m*a*t) we combine a final noise represented by *t*, the word *a+t* is produced; if to this simple combination we add various initial noises, several words are formed, as: *b*+at, *c*+at, *f*+at, *h*+at, *m*+at, *p*+at, *r*+at, *s*+at, *t*+at, *v*+at. However, the study of noises may well be passed until we understand the simpler and more interesting musical tones.

Tones are sounds having such continuity and definiteness that their characteristics may be appreciated by the ear, thus rendering them useful for musical purposes; these characteristics are *pitch* or *frequency*, *loudness* or *intensity*, and *quality* or *tone color*.

LECTURE II

CHARACTERISTICS OF TONES

Pitch

THE *pitch* of a sound is that tone characteristic of being acute or grave which determines its position in the musical scale; an acute sound is of high pitch, a grave sound is of low pitch. Experiment proves that pitch depends upon a very simple condition, the number of complete vibrations per second; this number is called the *frequency* of the vibration.

FIG. 24. Serrated disk for demonstrating the dependence of pitch upon frequency of vibration.

One of the simplest methods of determining pitch is mechanically to create vibrations at a rate which is known and which can be varied as desired; the rate is adjusted until the resulting sound is in unison with the one to be measured, then the number of vibrations generated by the machine is the same as that of the sound.

If a card is held against the serrated edge of a revolving disk, Fig. 24, the pulsations of the card produce vibrations in the air, and give rise to an unpleasant semi-musical sound, having a recognizable pitch which is measured by the number of taps given to the card per second. The four disks shown in the illustration have numbers of teeth in the ratios of 4 : 5 : 6 : 8, sounding the common chord, the pitch of which is dependent upon the rate of rotation of the disk.

FIG. 25. The siren.

The siren is an instrument in which the vibrations are produced by interrupting a jet of compressed air by means of a revolving disk with holes, as illustrated in Fig. 25; the sound is much softer and more musical than that from the serrated disk. The siren has been developed into an instrument suitable for research, Fig. 26, which enables one in a few minutes of time to determine to one part in a hundred the number of vibrations of common musical sounds. For securing greater range or for sounding several tones simultaneously, the siren is usually provided with two disks, d_1 and d_2, each having four rows of holes; one or more rows may be used at the same time, each producing its own pitch.[5] The disks may be rotated by compressed air on the principle of the turbine, or by an electric motor, as shown in the illustration; in either case the speed can be controlled, and the number of vibrations is determined with the aid of the revolution counter c between the two disks.

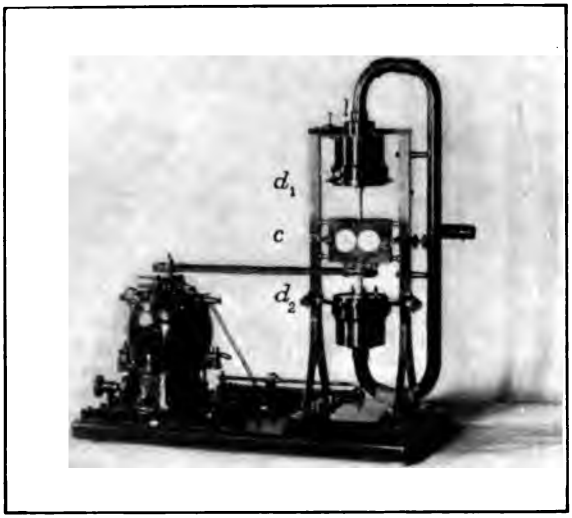


FIG. 26. Siren for the determination of pitch.

There are various other methods for determining and comparing the number of vibrations of sounding bodies, which are described in the references.[6] For the present purpose it will be sufficient to explain those used for the more precise determinations of a fundamental nature, the method of beats, Lissajous's optical method, and the methods of the clock-fork and the phonodeik.

The Tuning Fork

Perhaps the most important of acoustical instruments is the tuning fork invented in 1711 by John Shore, Handel's trumpeter. The fork reached an almost perfect development under the exquisite workmanship and painstaking research of Rudolph Koenig of Paris. When properly constructed and mounted, it gives tones of great purity and constancy of pitch; it is of very great value in experimental

Fig. 27. Tuning forks of various types.

work and provides the almost universal method of indicating and preserving standard pitches for all purposes.[7] Fig. 27 shows various forms of tuning forks, while Fig. 42 represents a larger collection, and many special forms are shown in other illustrations.

A tuning fork for scientific purposes should be made of one piece of cast steel, not hardened; the shapes developed by Koenig have not been excelled; the patterns for forks of ordinary musical pitches and those of very high pitches giving loud tones are shown in Fig. 28; a fork for $A_3 = 435$, of the first shape, is 129 millimeters long, not including the handle; a fork of the second shape, 79 millimeters long, has the pitch 3328.

The number of vibrations of a fork is dependent upon the mass of the prongs and the elastic forces due principally to the yoke; if the prongs are made lighter, by filing on the ends or sides, the pitch is raised; if the fork is filed near the yoke, the elastic restoring force is diminished and the pitch is lowered. The second shape of fork shown in the figure has a yoke which is very thick in proportion to the prongs, hence it is suitable for high pitches. A standard fork, having been accurately machined and finished, should be left with the prongs a trifle too long, that is, flat in pitch; the final tuning should be carried out very carefully by shortening both prongs together till the desired frequency is secured.

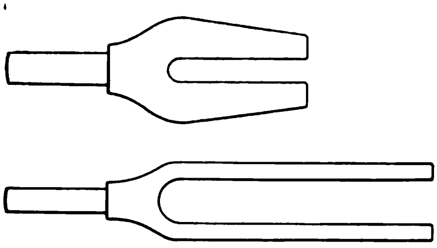


Fig. 28. Shapes of Koenig's tuning forks.

Filing or grinding a fork will heat it, as will also the touch of the fingers; the heating lowers the pitch of the fork, and if it is tuned while thus heated, it will later be found too sharp, that is, the prongs are already too short. Therefore the filing should stop while the fork is yet two or three tenths of a vibration flat, and the fork should be allowed to remain at a uniform temperature for a day or two before a comparison is made; if further tuning is necessary, it must be done with extreme care, and a comparison again made after another interval of rest. Vigorous filing will produce molecular disturbances which subside only after long periods of rest. The methods of comparison are described in the succeeding articles.

Tuning forks are often finished with a bright steel surface, in which case care is required to prevent rust; smearing with vaseline is a convenient rust preventive. A blued steel finish is excellent; standard forks are sometimes blued over the entire surface after all machine work has been finished, but before the final tuning; the final adjustment of pitch is made by careful grinding on the ends of the prongs, which are thus made bright, and the surfaces are then very lightly etched with a seal. Any further alteration of the fork, or an injury, will disfigure it and will be easily detected.

The boxes on which the forks are commonly mounted were first used by Marloye; they are of such dimensions that they form resonance chambers not quite in tune with the fork tone; if the tuning is perfect, the sound is louder but of short duration, because the energy of the vibration is more rapidly dissipated.[8] The box serves a double purpose: it produces a louder sound and it also purifies the tone by reinforcing only the fundamental. When the resonance box is not exactly in tune with the fork, it draws the fork out of its natural frequency by a small amount, a few thousandths of a vibration per second. These effects are considered at greater length under Resonance in Lecture VI.

Koenig proved that change of temperature alters the number of vibrations of a fork; the temperature coefficient was found to be nearly constant for forks of all pitches and to have the value -0.00011.[9] The change in the number of vibrations of a fork is found by multiplying its frequency by this coefficient and by the number of degrees of temperature change; the negative sign means that the frequency is diminished by increased temperature. For instance, a fork giving 435 vibrations per second at 15° C. will have its frequency diminished by 0.48 vibration for ten degrees increase in temperature.

The pitch of a fork changes slightly with the amplitude, that is, with the loudness of the tone which the fork is giving; the greater the amplitude, the less the frequency. For extreme changes in amplitude, the number of vibrations may vary as much as one in three hundred. If a fork is sounded loudly, the pitch will rise slightly as the tone subsides; the true pitch is that corresponding to a small intensity.[3]

Tuning forks may be excited by bowing across the end of one prong with a violin or a bass bow; this is perhaps the best method for obtaining the loudest possible response. For usual experimental work the most convenient method is to strike the fork with a soft hammer; a felt piano hammer head with a flexible spring handle is an excellent tool for the purpose; a solid rubber ball or a rubber stopper is often used for the hammer head. For sounding the thick high-pitched forks, an ivory hammer is best. Forks should never be struck with metal or other hard substances, for being of soft steel, they are likely to be injured.

Forks are often made to sound continuously by means of an electro-magnetic driving arrangement; a fork may be driven by itself, its own vibrations, once started, serving to produce the interrupted current required; such a fork is shown in Fig. 50, page 65. Often a fork is driven by an interrupted, or by an alternating, current produced from some other source; the ten forks shown in Fig. 179 are all driven by one interrupter fork at the back of the apparatus. In this instance the periods of the forks are exact multiples of that of the interrupter, since a fork will respond only to impulses which are in step with its own natural vibrations.

When a fork is driven by this method, the prong is intermittently urged forward by the magnetic pull. The prong itself is always a very little behind the pull, that is, it lags more or less; this forcing of the vibration causes the period to be slightly different from that of the same fork vibrating freely.[11]

A fork retains its pitch with great constancy; ordinary careless handling causes little change, and even rust, as it slowly proceeds over a period of years, produces but slight effect, rarely exceeding one vibration in two hundred and fifty; the change usually flattens the pitch, since rust near the yoke affects the fork more than that near the end of the prong. The ordinary wear on a fork is usually greater at the ends which are unprotected, and this causes the pitch to sharpen; rust and wear, then, in some degree produce opposite effects and tend to maintain the original pitch.

An account of the tone quality of the tuning fork is given in Lecture VI, while many illustrations of its usefulness will be found throughout the lectures.

Determination of Pitch by the Method of Beats

A simple comparison by the ear will enable one who is musically trained to tune certain intervals, such as unisons, octaves, thirds, fourths, and fifths. Two tones nearly in unison produce *beats*, the number of which per second is equal to the difference in pitch (see page 183). Beats often occur between the overtones of sounds which are not simple, and under other conditions which need not be considered here. Comparison by ear, based on the method of beats, is the principal means employed in tuning pianos and organs and such stringed instruments as the violin and the guitar.

The comparison of a standard tuning fork with an unknown pitch of nearly the same frequency can be made with ease and precision by the method of beats. The unknown sound and that of the standard fork being heard simultaneously, the number of beats per second is determined by counting the number occurring in five or ten seconds; the required pitch is then that of the standard increased or diminished by the number of beats per second. Usually the ear will decide whether the sound is flatter or sharper than the standard; in other cases it may be possible to make an easy adjustment to assist in this determination. When the sound is from a tuning fork, one prong may be loaded with a small piece of wax, which will slightly lower its pitch; if there are now more beats per second, the fork is flat, since making it flatter puts it further out of tune, and vice versa. The fork may be adjusted to equality with the standard by filing, as already explained, till the beats become fewer and finally cease.

When the two sounds approach unison, the interval between beats becomes longer; when the beats are slow, it is difficult to measure the time between them, for one is not sure of the instant of minimum or maximum sound. It is found that one can count beats with accuracy at the rate of from two to five per second, the count being carried over five or ten seconds, or more; four beats per second is perhaps the most convenient number. For these reasons an auxiliary fork is often used, which is tuned four beats per second sharper than the standard; the fork being tested is then adjusted till it is four beats per second flatter than the auxiliary, when it is, of course, exactly in unison with the standard.

Sets of forks are made for setting or testing the chromatic scale of equal temperament, as in tuning pianos and organs, in which the comparisons are made by beats. A series of thirteen forks is accurately tuned to the chromatic scale from middle C to C an octave higher; an auxiliary set of thirteen forks is then tuned so that each is exactly four beats per second sharper than the corresponding fork of the first series; a correctly tuned octave must have its successive tones four beats per second flatter than those of the auxiliary

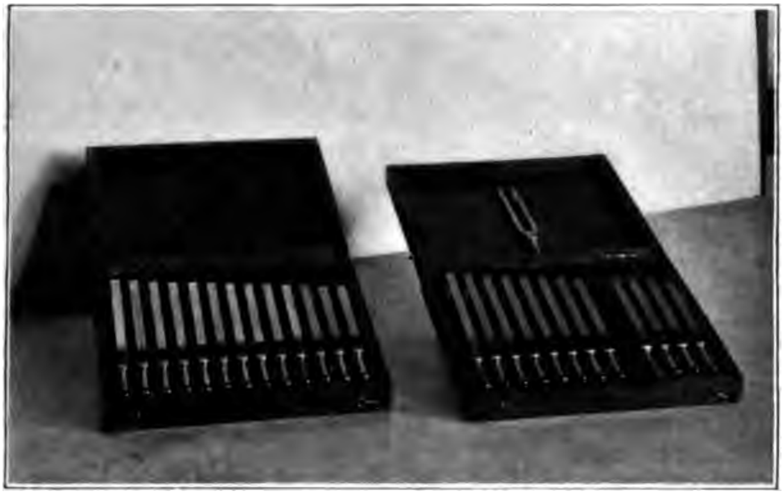


FIG. 29. Sets of forks for testing the accuracy of tuning the chromatic scale.

forks. Such forks are shown in Fig. 29; for making the tests the auxiliary forks only are actually required, but it is desirable to have the others also. The first and last forks of the scale set, which are an octave apart, give 258.65 and 517.3 vibrations, respectively, for A = 435; the auxiliary forks being four vibrations sharp give 262.65 and 521.3 vibrations, and are not a true octave apart; for a true octave the higher fork would be eight vibrations sharp and give 525.3 vibrations; none of the auxiliary forks gives true musical intervals.

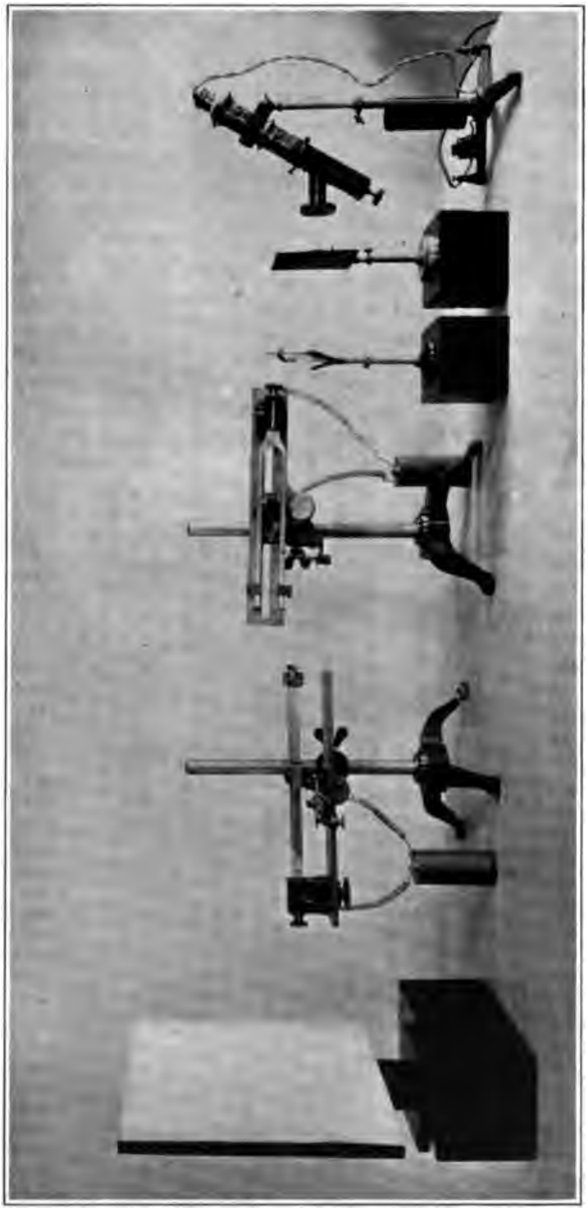


FIG. 30. Apparatus for projecting Lissajous's figures.

The absolute number of vibrations may be determined by counting the number of beats between the successive forks of a series of fifty or more ranging over one octave, according to the method devised by Scheibler in 1834.[12] Scheibler's *tonometer* consisted of fifty-six forks having pitches from about 220 to 440, the successive forks differing by four vibrations per second.

This method, which is very laborious, has been used by Ellis and by Koenig. Koenig's masterpiece is perhaps a tonometer consisting of a hundred and fifty forks of exquisite workmanship, and tuned with the greatest care and skill; it covers the entire range of audible sounds from 16 to 21,845.3 vibrations per second.[13] The largest fork is about five feet long, and has a cylindrical resonator eight feet in length and twenty inches in diameter. It is possible to find in this series a fork which shall differ from any given musical tone by not more than four beats per second, a comparison with which by the method of beats will determine the pitch of the sound with great ease and precision.

Optical Comparison of Pitches

One of the most precise methods for the comparison of frequencies is Lissajous's optical method,[14] which depends upon the geometrical figures traced by two simple harmonic motions at right angles. The motions may be provided by tuning forks which carry mirrors on the prongs, as shown in Fig. 30. A ray of light is reflected from one fork to the other and then to a screen or an observing telescope. When the forks are vibrating, the ray is deflected in two directions, so that the figure on the screen corresponds to the compounded motion. The shape of this figure is characteristic of the ratio of the frequencies of the two forks; for certain simple ratios, such as 1 : 2, 1 : 3, 2 : 3, etc., the figures are easily recognized by the eye; and when the ratio is exact, the figure exactly retraces itself, and because of the persistence of vision it appears continuous and stationary. If the ratio of frequencies is not exact, the figure changes, because of progressive phase difference, and, passing through a cycle, returns to the original form; the time for this cyclic change is that required for one fork to gain or lose one complete vibration on the exact number corresponding to the indicated ratio. The application of this method is explained in connection with the clock-fork.

The Clock-Fork

The most precise determinations of absolute pitch are those made by Koenig, who investigated the influence of the resonance box and of temperature on the frequency of a standard fork. He also determined the frequency of the forks used by the Conservatory of Music and the Grand Opera in Paris.[15] By combining the clock-fork of Niaudet with a vibration microscope for observing Lissajous's figures,[16] he developed the beautiful instrument shown in Fig. 31. Fig. 31 is reproduced from an autographed photograph of the original instrument, in the author's possession, while the instrument which was exhibited in the lecture is of more recent construction and is shown in Fig. 32, on page 40.

The apparatus is essentially a pendulum clock in which the ordinary pendulum is replaced by a tuning fork; the fork has a frequency of 64, as scientifically defined; that is, it makes 128 swings per second, counting both to and fro movements. The clock has the usual hour, minute, and second hands; but instead of the escapement operating on the second hand to release it once a second, the gearing of the movement is carried one step higher, and a fourth hand is provided, which goes round once in a second. A very small escapement mechanism is attached to this hand and is so arranged that it is operated by one prong of the tuning fork as it swings to and fro, 128 times a second; the fork thus releases the wheels regularly, as does an ordinary pendulum, and the clock "runs." Moreover, as in the common clock, the escapement not only releases the wheelwork, but it also imparts a small impulse to the fork so as to maintain its vibration as long as the clock runs; that is, for days if desired. Thus we have a tuning fork which will vibrate continuously, and a clock-work which accurately counts the vibrations.

FIG. 31. Photograph of Koenig's clock-fork bearing his autograph.

The rate of the fork is adjusted much as is a pendulum, by moving small weights up or down on threaded supports. If the clock is regulated till it keeps correct time, the fork must vibrate exactly 128 times a second, making 11,059,200 single vibrations in a day. A change in the rate of the clock of one second per day means a change in the frequency of the fork of one part in eighty-six thousand four hundred; that is, when the clock loses one second a day, the fork has a frequency of 63.99926. If it is desired, for instance, to adjust the fork to exactly 63 complete vibrations per second, the clock must lose one part in sixty-four, that is, it must lose 22½ minutes a day.

By means of the weights the actual fork can be adjusted to have any desired frequency between 62 and 68, this frequency being determined to a ten-thousandth part of a

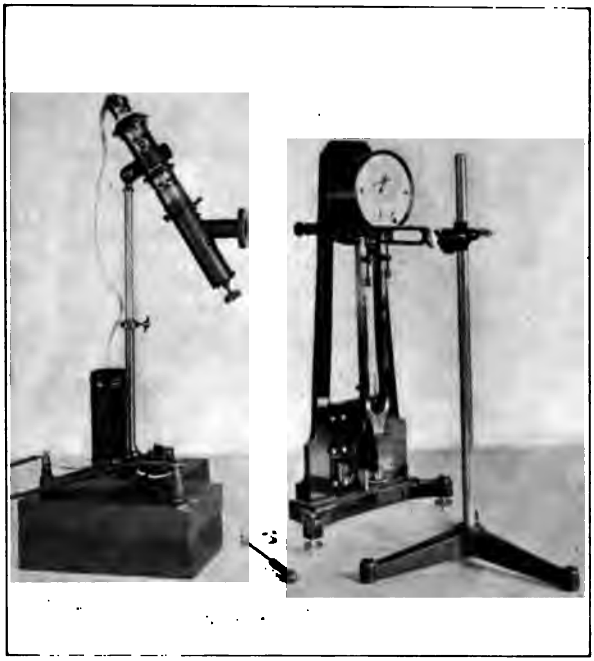


Fig. 32. Clock-fork arranged for verifying another fork.

vibration by the rate at which the clock gains or loses. The range of the fork is a musical semi-tone; by using various multiples of its frequency, it is possible to determine almost any desired musical pitch with precision.

The method of using the clock-fork may be illustrated by a concrete example; thus the verification of a standard A = 435 fork requires the following procedure. The only integral divisor of 435 which will give a quotient within the limits of frequency of the clock-fork is 7, and the quotient is 62.143; a numerical calculation shows that if the clock loses 41 minutes 46 seconds per day, or 1 minute 44.4 seconds per hour, the tuning-fork pendulum will make 62.143 vibrations per second; if the A-fork vibrates exactly 7 times as fast, its frequency must be 435; the exact ratio of the frequencies is to be determined by Lissajous's figures with the vibration microscope.

The clock-fork carries the objective lens of a microscope, the body of which is attached to the frame. The A-fork is supported so that its line of vibration is at right angles to that of the lens, Fig. 32, and so that some brightly illuminated point, as a speck of chalk dust on the end of the prong, is visible through the microscope; if the two forks are vibrating, this speck is seen to describe the Lissajous curve for the ratio of 1 : 7, Fig. 33. Suppose now the figure goes through its cyclic change once in 5 seconds, then the fork has a frequency of either 434.8, or 435.2. To determine whether the fork is sharp or flat a very small piece of wax is attached to one prong, which will make it vibrate more slowly. If the cyclic change requires a longer time than before, the slight lowering of pitch has improved the tuning, which condition indicates that the frequency of the fork was 435.2; if the change occurs in less time, the lowering of the pitch has made it further from the true value and the fork had a frequency of 434.8. The A-fork may be adjusted by filing or grinding, near the ends of the prongs to make it sharper or near the yoke to make it flatter, as described on page 30, and the adjustment may be continued till any

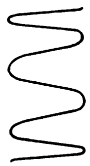


FIG. 33. Lissajous's figure for the ratio 1 : 7.

required accuracy has been obtained; for instance, if the cyclic change occurs in 10 seconds, the error of tuning is $\frac{1}{10}$ vibration per second.

The clock-fork is provided with a mirror on the side of one prong so that it may be used to produce Lissajous's figures by the light-ray method or to record the vibrations directly on a photographic film.

Pitch Limits

The range of pitch for the human voice in singing is from 60 for a low bass voice to about 1300 for a very high soprano.

Fig. 34. Organ pipe over 32 feet long giving 16 vibrations per second.

The piano has a range of pitch from 27.2 to 4138.4. The pipe organ usually has 16 for the lowest pitch and 4138 for the highest; an organ pipe giving 16 vibrations per second, Fig. 34, is nominally 32 feet long, though its actual length is somewhat greater; there are a few organs in the world having pipes 64 feet long which give only 8 vibrations per second, but such a sound is hardly to be classed as a musical tone; the frequency 4138 is given by a pipe $1\frac{1}{2}$ inches long.

Neither speech nor music makes direct use of all the sounds which the ear can hear. Helmholtz considered 32 vibrations per second as the lowest limit for a musical sound, that is, one which gives the sensation of a continuous tone; yet the piano descends to 27 and the organ to 16 or even to 8 vibrations per second. The tuning fork shown in Fig. 35

may be made to give from 16 to 32 vibrations per second, according to the position of the weights on the prongs. Experimenters differ widely as to the lower limit, though nearly all consider Helmholtz's value too high; perhaps the most trustworthy values are between 12 and 20 vibrations per second, with a general consensus of opinion that the lower limit of audibility for a musical tone is 16 vibrations per second. Of course, the ear can hear vibrations when they are fewer in number than 16 per second, but they are heard as separated or discontinuous sounds.

FIG. 35. Large fork giving from 16 to 32 vibrations per second.

It is interesting to notice that the frequency of repetition of an impression to produce continuity of sensation for sound is practically the same as for light. The persistence of vision is about one tenth of a second, that is, an intermittent visual sensation occurring ten times or more a second produces the effect of a continuous sensation; for moving pictures the views are usually changed sixteen times a second, and the intermittent movement, or vibration, at this rate, gives the impression of a continuous motion. If a screen is illuminated with a moving-picture projection apparatus in which there is no picture, the eye perceives a flicker in the general illumination when the intermittent shutter of the machine is in operation, unless the number of light flashes per second exceeds a certain value.[17] This value varies from ten to fifty or more per second, according to the intensity of the light. Perhaps Helmholtz's value of 32 for the lower limit of a tone is the flicker limit for the ear.

While the upper pitch limit for the musical scale is about 4138, the ear can hear sounds having frequencies of 20,000

Fig. 36. Small organ pipe giving 15,600 vibrations per second.

or 30,000, and even more in cases of extreme sensitiveness. Fig. 36 shows what is in form a regular organ pipe, one of the smallest ever made, and much too small to be used in an organ; the length of the pipe which is effective in producing the tone is indicated by *l* in the figure and measures 0.25 inch. This pipe sounds B_8 and gives 15,600 complete vibrations per second, a sound which is clearly audible to most listeners. Experiments to determine the upper limit of audibility are often made with a Galton's whistle, Fig. 37, an adjustable whistle or stopped organ pipe of very small dimensions, blown by means of a rubber pressure bulb. The whistle can be set to various lengths, indicated by the graduated scales, giving high-pitched sounds of known frequency. Another experimental method of producing sounds of high pitch is by the longitudinal vibration of short steel bars, Fig. 38. The bars are suspended by silk cords,

FIG. 37. Adjustable whistle for determining the frequency of the highest audible sound.

and are struck on the ends with a steel hammer, producing a clear metallic ringing sound, which is the tone desired; the pitch of the sound is determined by the length of the bar, a bar 52.5 millimeters ($2\frac{1}{16}$ inches) long giving 32,768 vibrations per second.

Perhaps the most conclusive experiments on audible and inaudible tones of the highest pitch are those of Koenig, extending over a lifetime of investigation,[18] in which observations were made with tuning forks, transverse vibrations of rods, longitudinal vibrations of rods, plates, organ pipes, membranes, and strings. Tuning forks, when properly constructed and used, proved to be the most suitable source of high tones. Koenig made his first experiments in 1874 when he was forty-one years old and at that time was able to hear tones up to $F_9\sharp = 23{,}000$, which he considered the highest directly audible simple tone; he constructed a set of forks up to $F_9 = 21{,}845$, which he exhibited at the Centennial Exposition in Philadelphia in 1876. (These forks and much other

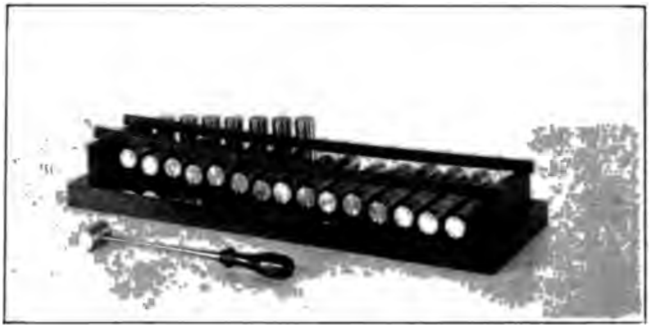


FIG. 38. Steel bars for testing the highest audible frequency of vibration.

interesting acoustic apparatus exhibited by Koenig are now in the laboratory of Toronto University.) In his fifty-seventh year the limit of audibility for Koenig was $E_9 = 20{,}480$, and in his sixty-seventh year it was $D_9\sharp = 18{,}432$. A set of Koenig forks for tones of high pitch is shown in Fig. 39; Koenig has made a complete series of such forks extending more than two octaves above the limit of audibility to a frequency of 90,000 complete vibrations (180,000 motions to and fro) per second. Sounds which are inaudible are made evident by cork-dust figures in a tube, Fig. 39; the stationary air waves produced by the vibration of the fork at the end of the tube cause the cork dust to accumulate in little heaps, one in each half wave length of the sound. The wave length in air for the tone of 90,000 frequency is 1.9 millimeters, or 0.075 inch.

Though the pitch of the highest note commonly used in music is 4138, overtones with frequencies of 10,000, or more, probably enter into the composition of some of the sounds of music and speech. The investigation of these tones of very high pitch should not be neglected; however, the

FIG. 39. Forks for testing the highest audible frequency of vibration.

analytical work discussed is these lectures in limited to pitches of from about 100 to 5000.

While it would be interesting to students of music to consider the reasons for the selection of tones of certain pitches to form scales and chords, it would lead us far from our present purpose; it will, however, be useful to notice the location on the musical staff of the octave points of the sounds used in music and to explain the notation which designates a given tone. The musical staff may be considered as composed of eleven lines; to assist in identifying

TABLE OF EQUALLY TEMPERED SCALE, $A_3 = 435$

	C_{-1}–C_0	C_0–C_1	C_1–C_2	C_2–C_3	C_3–C_4	C_4–C_5	C_5–C_6	C_6–C_7
C	16.17	32.33	64.66	129.33	258.65	517.31	1034.61	2069.22
C♯	17.13	34.25	68.51	137.02	274.03	548.07	1096.13	2192.26
D	18.15	36.29	72.58	145.16	290.33	580.66	1161.31	2322.62
D♯	19.22	38.45	76.90	153.80	307.59	615.18	1230.37	2460.73
E	20.37	40.74	81.47	162.94	325.88	651.76	1303.53	2607.05
F	21.58	43.16	86.31	172.63	345.26	690.52	1381.04	2762.08
F♯	22.86	45.72	91.45	182.89	365.79	731.58	1463.16	2926.32
G	24.22	48.44	96.89	193.77	387.54	775.08	1550.16	3100.33
G♯	25.66	51.32	102.65	205.29	410.59	821.17	1642.34	3284.68
A	27.19	54.37	108.75	217.50	**435.00**	870.00	1740.00	3480.00
A♯	28.80	57.61	115.22	230.43	460.87	921.73	1843.47	3686.93
B	30.52	61.03	122.07	244.14	488.27	976.54	1953.08	3906.17
C	32.33	64.66	129.33	258.65	517.31	1034.61	2069.22	4138.44

the lines, the middle one is omitted except when required for a note, Fig. 40; additional lines of short length are used

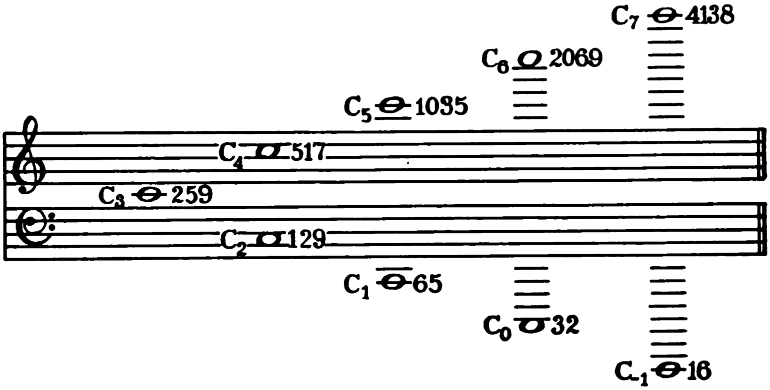


FIG. 40. Middle C and the several octaves of the musical scale.

to extend the compass. The tone called "middle C" is placed on the line between the bass and treble staffs, and is designated by C_3; in International Pitch this tone has 258.65 vibrations per second; the musical compass is four octaves upward and downward from middle C, the various octaves bearing subscripts as shown; all the tones of an octave between two C's are designated by the subscript of the lower C; that is, G_3 is on the second line of the treble staff, and G_1 is on the lowest line of the bass staff, etc.

The table opposite gives the pitch numbers for all the tones of the equally tempered musical scale, based on International Pitch, $A_3 = 435$.

Standard Pitches

Musical pitch is usually specified by giving the number of vibrations of the note called "Violin A," , though sometimes it is given by "Middle C," , or by the C an octave higher. The standard of musical pitch has varied greatly, even within the history of modern music, from the classical pitch of the time of Handel and Mozart, when it was A = 422, to the modern American Concert pitch of A = 461.6, a change of more than one and a half semi-tones. Ellis gives a table of two hundred and forty-two pitches, showing values for A ranging from 370 to 567, that is, from F♯ to D of the modern musical scale.[19] The conditions of use and cause of changes in pitch are described in the references. Especially interesting are the accounts of the changes in Philharmonic Pitch, that of the London Philharmonic Orchestra, which under Sir George Smart, in 1826, was A = 433, and under Sir Michael Costa, in 1845, was raised to A = 455. In America,[20] the equivalent of this Philharmonic Pitch is often referred to as Concert Pitch, and it has reached the high limit of A = 461.6. Not only has the rise in pitch been so great that artists have refused to sing and instrument strings frequently break under the strain, but the lack of uniformity also causes great confusion and trouble.

A convention of physicists in Stuttgart in 1834 adopted Scheibler's pitch of A = 440, which has been much used in Germany; this is perhaps the first standard pitch.

As a result of Koenig's researches with the clock-fork, the French "Diapason Normal," A = 435 at the temperature of 20° C., was established in 1859. This was adopted by several of the leading symphony and opera orchestras; the Boston Symphony Orchestra adopted this pitch upon its organization in 1883.

A committee of the Piano Manufacturers' Association of America, of which General Levi K. Fuller was chairman, made an extensive investigation of musical pitch, assisted by Professor Charles R. Cross of Massachusetts Institute of Technology. After consultation with many authorities in this country and Europe, the Committee, in 1891, adopted as the standard the Diapason Normal as determined by Koenig and named it "International Pitch, A = 435," at a temperature of 20° C. (68° F.). This is often called Low Pitch in distinction from Concert or Philharmonic Pitch, which is now referred to as High Pitch. The committee selected as its fundamental standard the type of fork made by Koenig, shown in Fig. 41, which is provided with an adjustable cylindrical resonator and gives a tone of great strength and purity.

It has been proposed that A = 438 be made a standard, as a compromise between the Stuttgart A = 440 and the

Diapason Normal A = 435; for practical purposes there is little difference in the pitches 435, 438, and 440; but there should be but one nominal standard, and it seems that the strongest arguments favor the universal adoption of A = 435. The musician should insist that his piano and other instruments be tuned to this pitch.

Fig. 41. Standard fork. International Pitch. A = 435.

Before any standard had been generally established for musical purposes, Koenig adopted one for his own work, and as tuning forks of his make are widely used in scientific institutions, this pitch, in which middle C = 256, is often referred to as Scientific or Philosophical Pitch.

The author urges the use of one pitch only for both scientific and musical purposes, viz. A = 435; in the tempered

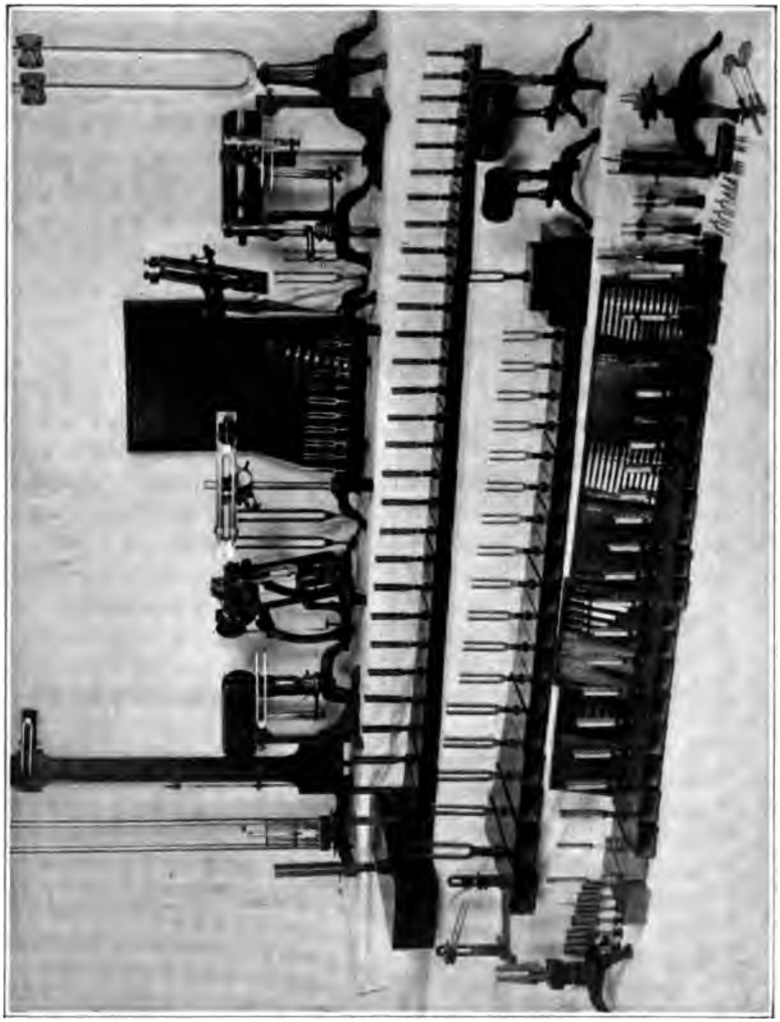


FIG. 42. A collection of tuning forks.

musical scale this gives for middle C 258.65 vibrations per second. This pitch is used exclusively in discussing the results of our sound analysis. In the laboratory of Case School of Applied Science the scale forks based on C = 256 have been duplicated with new forks based on A = 435; Fig. 42 shows the larger part of this collection, there being over two hundred forks in the picture.

Intensity and Loudness

The *loudness* of a sound is a comparative statement of the strength of the sensation received through the ear. It is impossible to state simply the factors determining loudness. For the corresponding characteristic of light (illumination) there is a moderately definite standard, commonly called the candle power; but for sound there is no available unit of loudness, and we are dependent on the subjective comparison of our sensations.[21] Not only are the ears of different hearers of different sensitiveness, but each individual ear has a varying sensitiveness to sounds of different pitches and, therefore, to sounds of various tone colors.

In a first study of the physical characteristics of sounds we are compelled to consider the intensity not as the loudness perceived by the ear, but as determined by what the physicist calls the energy of the vibration. Fortunately, under simple conditions and within the range of pitch of the more common sounds of speech and music, there is a reasonable correspondence between loudness and energy.

The *energy*, or what we will call the intensity of a simple vibratory motion, varies as the square of the amplitude, the frequency remaining constant; it varies as the square of the frequency, the amplitude remaining constant; when both amplitude and frequency vary, the intensity varies as the square of the product of amplitude and frequency; or to express it by a formula, representing intensity by I, amplitude by A, and frequency by n,

$$I = n^2A^2.$$

Since we are to study sounds by means of representative curves or wave lines, we may give attention to the features of the curves which indicate intensity. In Fig. 43 the curve

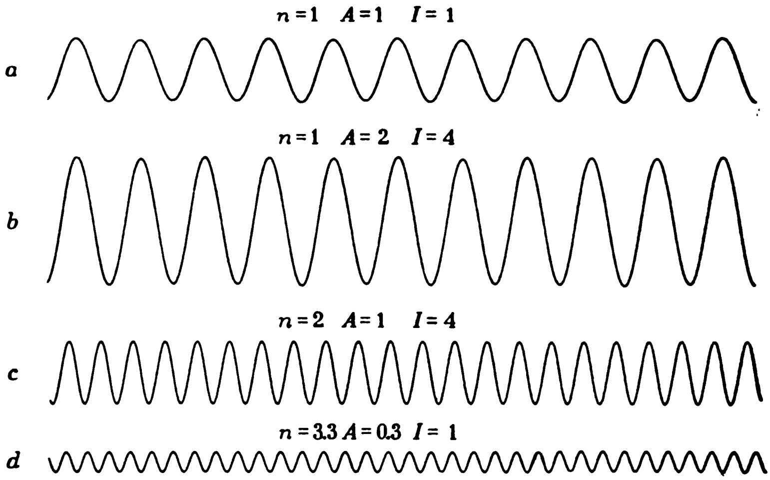


FIG. 43. Curves representing simple sounds of various degrees of loudness.

b has a frequency the same as that of curve *a*, but its amplitude is twice as great, hence it represents a sound four times as loud; the curve *c* has an amplitude the same as that of *a*, but its frequency is twice as great, and again its loudness is four times that of *a*; the curve *d* has a frequency of 3.3 and an amplitude of 0.3, and it represents a loudness equal to that of *a*. Then the sounds represented by *a* and *d* are of equal loudness; and those represented by *b* and *c* are equal, but are four times as loud as *a* or *d*.

Caution is necessary when making inferences from simple inspection of photographic records of sound vibrations, since a change of film speed may give an apparent change of frequency when none really exists.

When we are studying the records of complex sounds, and

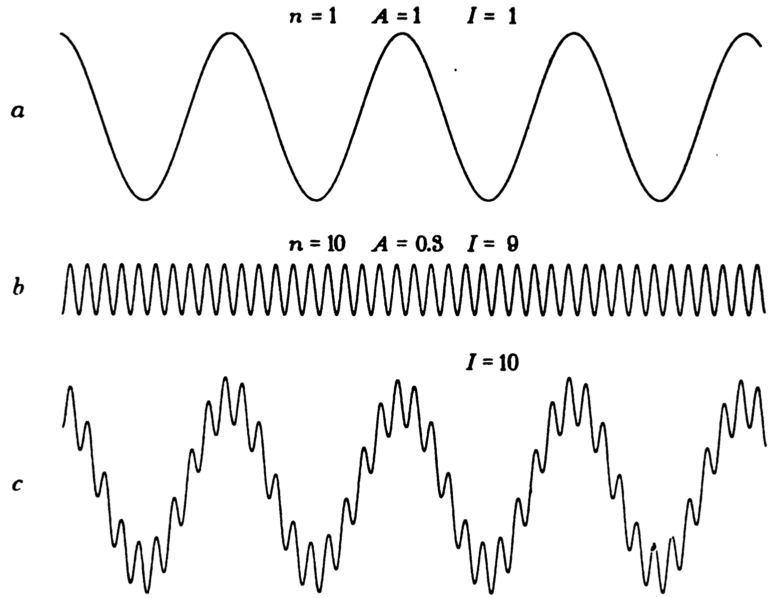


FIG. 44. Curves representing two simple sounds and their combination.

practically all sounds are such, a simple measurement of the amplitude of the curve and of the frequency is not sufficient for a determination of the loudness; it is necessary to analyze the wave into its simple components, to compute the intensity due to each component singly, and then to take the sum of these intensities; Fig. 44 illustrates this condition. Curves *a* and *b* have loudnesses represented by 1 and 9, as explained above; curve *c* contains both *a* and *b* and its true loudness is therefore 10. If it were assumed that the loudness of *a* and *c* are represented by the squares of their measured widths, the value for *c* would be 1.6 as compared with *a*, which is only one sixth of its real loudness.

A further illustration of the necessity for analysis of a wave before judging of the loudness is shown in Fig. 45, in which *a* and *b* are of exactly the same loudness though of different widths. The curve *a* is composed of two partials, a fundamental and its second overtone, of loudness 1 and 4, respectively; *b* is composed of the same partials, and therefore has the same loudness. The curves differ only in the relation of the phases of the components.

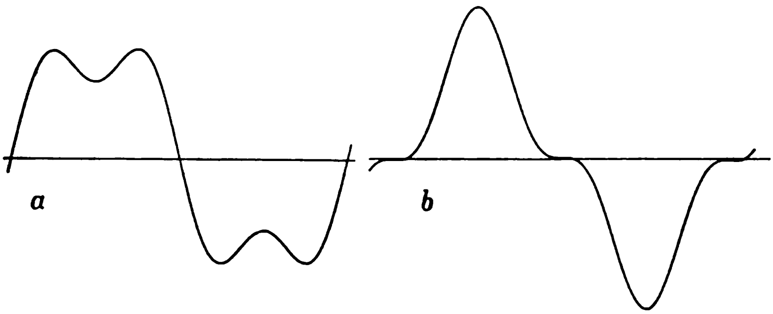


FIG. 45. Curves representing the combination of two simple sounds in different phases; though the curves are of different widths, they represent sounds of the same loudness.

ACOUSTIC PROPERTIES OF AUDITORIUMS

The loudness of a sound as perceived by the ear depends not only upon the characteristics of the source, but also upon the peculiarities of the surroundings. Among the features of an auditorium which must be considered are its size and shape, the materials of which it is constructed, its furnishings, including the audience, and the position of the source.

The determination of the acoustic properties of auditoriums is of the very greatest practical importance, and it is also one of the most elusive of problems; the sounds which most interest us are of short duration and they leave no trace, and the conditions affecting the production, the transmission, and the perception of sound are extremely complicated. The difficulties of the work are such as to discourage any but the most skillful and determined investigator. Indeed, the problem has been almost universally considered impossible of solution; and this opinion has been accepted with so much complacence, and even with satisfaction, that it still persists in spite of the fact that a scientific method of determining the acoustic properties of auditoriums has been developed by Professor Wallace C. Sabine of Harvard University. This method, which is of remarkable practical utility, has been described in architectural and scientific journals.[22] No auditorium, large or small, and no music room, public or private, should be constructed which is not designed in accordance with these principles. Sabine's experiments have shown that the most common defect of auditoriums is due to reverberation, a confusion and diffusion of sound throughout the room which obscures portions of speech. There are other effects, due to echoes, interferences, and reflection in general, all of which have been considered. In many cases these troubles can be remedied, with more or less difficulty, in auditoriums already constructed; this is especially true in regard to reverberation, which is reduced by the proper use of thick absorbing felt placed on the side walls and ceiling.

A method for photographing the progress of sound waves in an auditorium is referred to in Lecture III, page 88, which bears indirectly upon the loudness of the sound and is of great value in designing rooms which shall be free from defects.

A soundboard placed behind the speaker may, in some instances, distribute the sound in such a way as to remedy certain defects, as has been shown by the elaborate experiments of Professor Floyd R. Watson,[23] but the more common faults are not removed by this method. An auditorium has been described by Professor Frank P. Whitman, which was practically unimproved by the use of a soundboard, and was later made altogether satisfactory for public speaking upon the removal of reverberation by Sabine's method.[24]

It may be added that the stringing of wires or cords across an auditorium can in no degree whatever remove acoustical defects.

Tone Quality

The third property of tone is much the most complicated; it is that characteristic of sounds, produced by some particular instrument or voice, by which they are distinguished from sounds of the same loudness and pitch, produced by other instruments or voices. This characteristic may be called *tone color*, *tone quality*, or simply *quality*.

With comparatively little practice one can acquire the ability to recognize with ease any one of a series of musical instruments, when they produce tones of the same loudness and pitch. There is an almost infinite variety of tone quality; not only do different instruments have characteristic qualities, but individual instruments of the same family show delicate shades of tone quality; and even notes of the same pitch can be sounded on a single instrument with qualitative variations. The bowed instruments of the violin family possess this property in a marked degree.

No musical instrument equals the human voice in the ability to produce sounds of varied qualities; the different vowels are tones, each of a distinct musical quality. The investigation of tone quality therefore leads to a study of vocal as well as instrumental sounds.

Since pitch depends upon frequency, and loudness upon amplitude (and frequency), we conclude that quality must depend upon the only other property of a periodic vibration, the peculiar kind or form of the motion; or if we represent the vibration by a curve or wave line, quality is dependent upon the peculiarities represented by the shape of the curve.

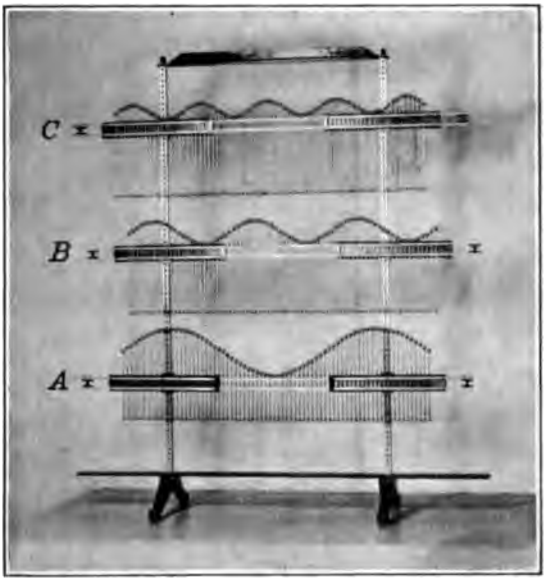


FIG. 46. Models of three simple waves, having frequencies in the ratios of 1 : 2 : 3.

The simplest possible type of vibration, simple harmonic motion, and its representative curve, the sine curve, were described in the preceding lecture. A tuning fork, when properly mounted on a resonance box, gives to the air a single simple harmonic motion, which, being propagated, develops a simple wave. The sensation of such a tone is absolutely simple and pure.

The nature of tone quality may be explained with the aid of tuning forks and the wave models [25] shown in Fig. 46.

Let one of the forks having the pitch C_3 be sounded; it will produce a simple wave in the air, which may be represented by the model *A*; a second fork, one octave higher, will, when sounding alone, send out twice as many vibrations per second, generating simple waves of just half the wave length, as represented by the model *B*; a third fork, vibrating three times as fast as the first, produces waves one third as long, shown by model *C*. These simple models illustrate two characteristics of tone: pitch, by the frequency or number of waves in a given length, and loudness, by the height or amplitude of the waves.

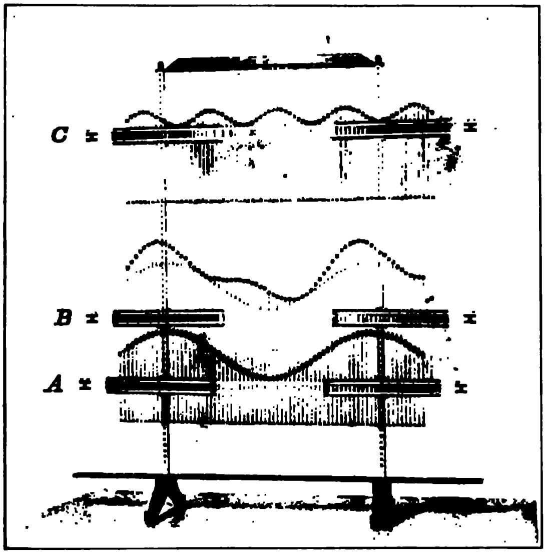


FIG. 47. Wave form resulting from the composition of two simple waves.

If two forks are sounded at the same time, the two corresponding simple motions must exist simultaneously in the air, and the motion of a single particle at any instant must be the algebraic sum of the motions due to each fork separately. This condition is shown in Fig. 47, where the wave *B* has been lowered to rest on the top of *A*, impressing the form of *A* upon *B*, which now exhibits the form of the motion due to the two simple sounds. When the three forks are sounding, the form of the composite motion is shown by lowering the wave form *C* upon that of *A* and *B*, as shown in Fig. 48.

The relative phase of a wave may be shifted by changing the position of one of the forks in relation to the others; this effect is demonstrated by shifting the corresponding wave form sidewise (in the direction of the length of the wave) before the forms are pushed together; the shape of the resulting wave is thus changed while its composition remains the same.

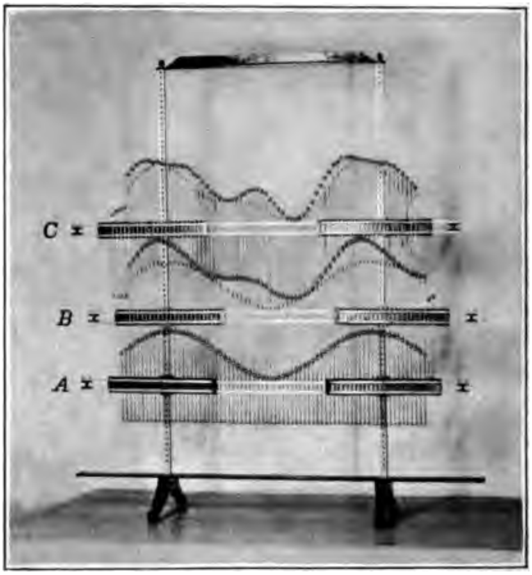


FIG. 48. Wave form resulting from the composition of three simple waves, corresponding to a composite sound containing three partials.

This argument may be extended indefinitely to include any number of simple tones of any selected frequencies, amplitudes, and phases. There are therefore peculiarities in the motion of a single particle of air which differ for a single tone and for a combination of tones; and in fact the kind of motion during any one period may be of infinite variety, corresponding to all possible tone qualities. These lectures are concerned almost wholly with the development and the application of this principle.

Law of Tone Quality

The law of tone quality was first definitely stated in 1843 by Ohm of Munich, in *Ohm's Law of Acoustics*, and much of Helmholtz's work of thirty years later was devoted to the elaboration and justification of this law.[26]

The law states: all musical tones are periodic; the human ear perceives pendular vibrations alone as simple tones; all varieties of tone quality are due to particular combinations of a larger or smaller number of simple tones; every motion of the air which corresponds to a complex musical tone or to a composite mass of musical tones is capable of being analyzed into a sum of simple pendular vibrations, and to each simple vibration corresponds a simple tone which the ear may hear.

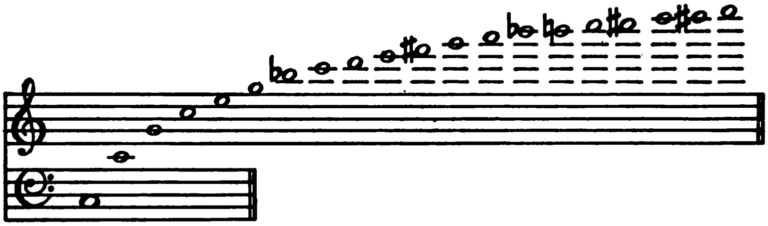
From this principle it follows that nearly all the sounds which we study are composites. The separate component tones are called *partial tones*, or simply *partials*; the partial having the lowest frequency is the *fundamental*, while the others are *overtones*. It sometimes happens that a partial not the lowest in frequency is so predominant that it may be mistaken for the fundamental, as with bells; and sometimes the pitch is characterized by a subjective beat-tone fundamental when no physical tone of this pitch exists. If the overtones have frequencies which are exact multiples of that of the fundamental they are often called *harmonics*, otherwise they may be designated as inharmonic partials.

As the result of elaborate investigation, Helmholtz added the following law: the quality of a musical tone depends solely on the number and relative strength of its partial simple tones, and in no respect on their differences of phase.[27] Koenig, after experimenting with the wave siren (Fig. 178, page 245), argued that phase relations do affect tone quality in some degree.[28] Lindig has used a "telephone-siren" and concludes that the phases of the components influence quality of tone only through interference effects.[29] Lloyd and Agnew, using special alternating current generators in connection with a telephone receiver, have found that the phase differences of the components do not affect the quality of tone.[30] The question has been extensively investigated by many others, with a consensus of opinion that Helmholtz's statement is justified.[31]

In the analysis of sound waves from instruments and voices, described in Lectures VI and VII, the phases of all component tones have been determined. While systematic study of the phases has not yet been made, no evidence has appeared which indicates that the phase relation of the partials has any effect upon the quality of the tone. If tone quality varies with phase relations, the variations certainly are very small in comparison with those due to other influences.

The analyses which have been made give abundant evidence that tone quality as perceived by the ear is much influenced by subjective beat-tones. While these tones may be considered as having no physical existence, yet their effects upon the ear are those of real partials, and the laws already stated include them. An explanation of beat-tones is given in Lecture VI, page 183.

Fig. 49 shows on the musical staff the relations of a fundamental tone, $C_2 = 129$, and nineteen of its harmonic overtones. The numerals in the line below the staff indicate the *orders* of the several partials. In the next lower line are given the frequencies of the partials when they are harmonic. Every sound which is represented by a periodic wave form must have harmonic overtones, as will be more fully explained in Lecture IV; such sounds are generally described as musical. The partial tones of sounds such as the clang of a bell are inharmonic and would not correspond to the scheme shown in the figure. The tones of the musical chromatic scale are determined according to the scheme of equal temperament developed by Bach. The various harmonic overtones of a given sound are not in tune with any notes of the musical scale, except such as are one or more



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
129	259	388	517	647	776	905	1035	1164	1293	1423	1552	1681	1811	1940	2069	2199	2328	2457	2586
C	C	G	C	E	G	B♭ C	D	E	G♭	G	G♯	B♭	B	C	C♯	D	D♯	E	
129	259	388	517	652	775	922	1035	1161	1304	1463	1550	1642	1843	1953	2069	2192	2323	2461	2607

FIG. 49. A fundamental and its harmonic overtones.

exact octaves from the fundamental. The notes on the staff in Fig. 49 represent the scale tones which are nearest to the overtones; the lower lines in the figure give the designations of the notes and their frequencies in the tempered scale.

Overtones can be illustrated by vibrating strings in such a way as to make their nature directly visible. A silk cord may be made to vibrate by a large electrically driven fork, as shown in Fig. 50, with the formation of a single loop due to vibration in the fundamental mode. By changing the tension of the string, it can be made to vibrate in various sub-

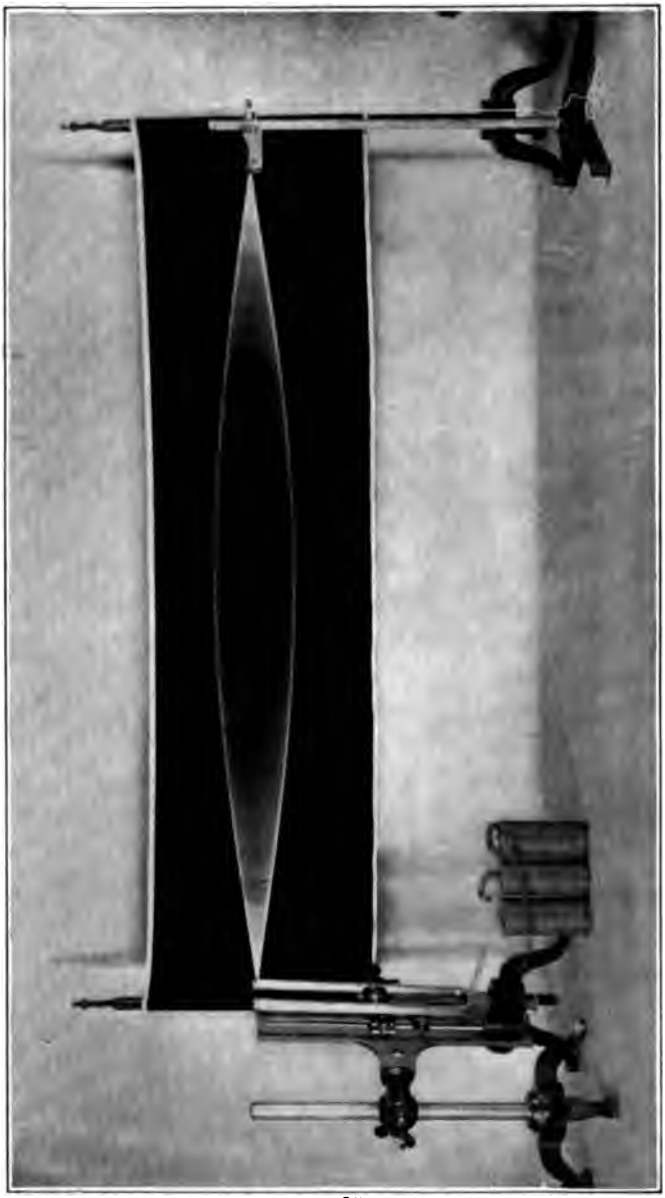


FIG. 50. A string vibrating in a single loop, corresponding to a simple tone consisting of a fundamental only.

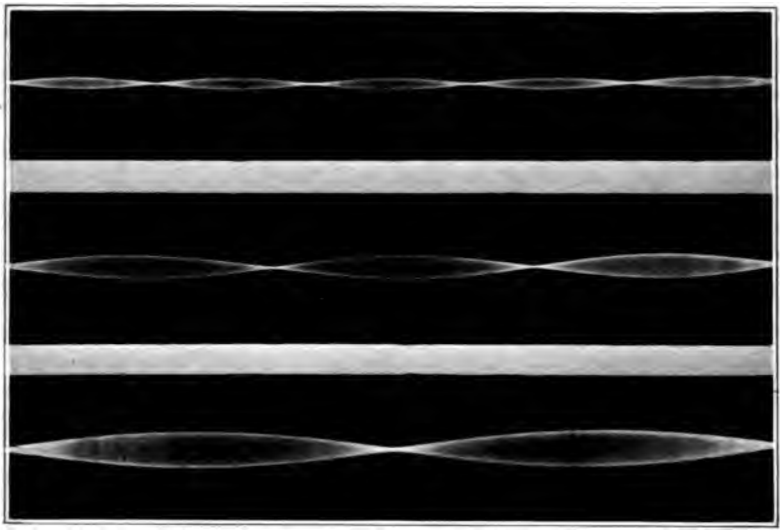


Fig. 51. Simple vibrations of a string in various subdivisions, corresponding to harmonic overtones or partials.

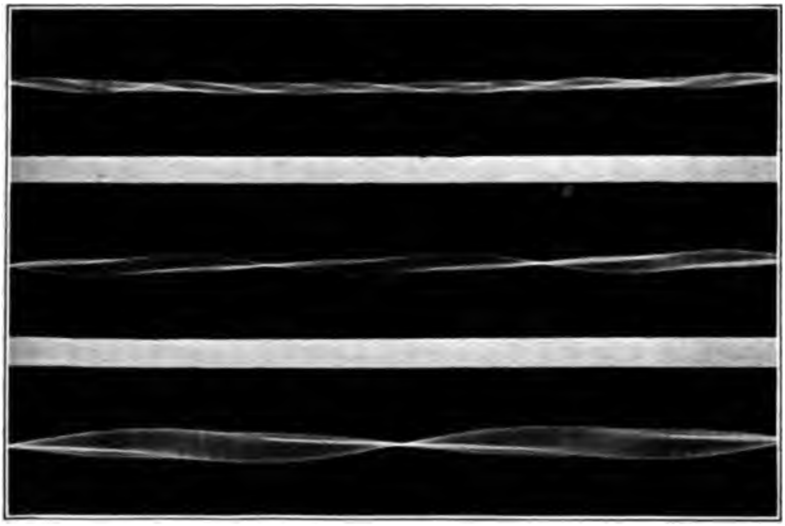


Fig. 52. Complex vibrations of a string, showing the coexistence of several modes of vibration, representing tones having different qualities.

divisions corresponding to its harmonic overtones; Fig. 51 shows two-loop, three-loop, and five-loop formations, representing the first, second, and fourth overtones. A string vibrating in these forms would emit simple tones only; if the pitch for the single loop is $C_2 = 129$, the two loops would correspond to the tone $C_3 = 259$, the three loops to $G_3 = 387$, and the five loops to $E_4 = 645$.

A string may be made to vibrate in complex modes, with the simultaneous existence of several loop formations. Fig. 52 shows the vibrations with two and four loops, with

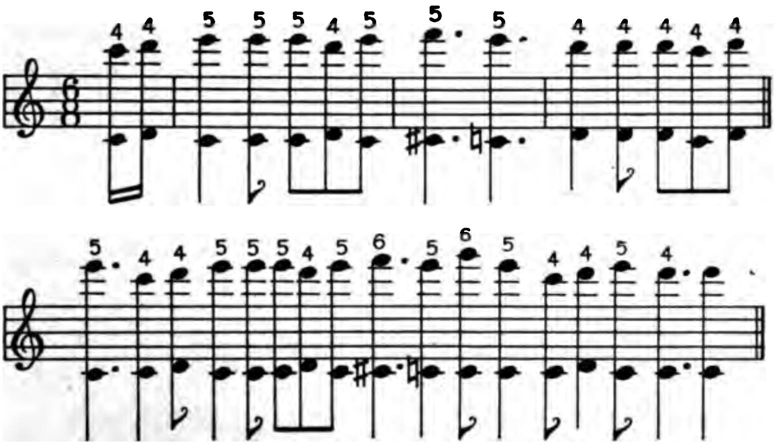


FIG. 53. A tune in harmonics.

three and six loops, and at the top a much more complex combination; these forms represent composite sounds, each set of loops corresponding to a partial tone.

The multiplicity of tones from one air column, corresponding to the several loop formations in a vibrating string, are illustrated by wind instruments, many of which use harmonic tones in their regular scales. The bugle can sound only tones due to the vibration of the air column in various subdivisions of its fundamental length; it produces the tones of the harmonic series shown in Fig. 49. A flute tube without holes or keys may be made to sound ten or more tones of the harmonic series. A tune can be played on a flute by using the harmonic tones of only three fundamentals, requiring two keys which are manipulated by one finger; the illustration, Fig. 53, shows at the bottom the notes fingered, while those at the top are the harmonic tones sounded; the small numerals indicate the orders of the partials used for the several tones.

Analysis by the Ear

Even after the arguments presented, it may seem strange that a single source of sound can emit several distinct tones simultaneously. There is, however, abundant experimental evidence in support of the statement. By listening attentively, one can often distinguish several component tones in the sound from a flute or violin or other instrument.

Fig. 54. Helmholtz resonators.

Helmholtz, who depended mainly upon the ear for the analysis of composite sounds, developed several methods for assisting the ear in the detection of partial tones.[32] He devised the tuned spherical *resonator* which he used with remarkable success. Fig. 54 shows a series of Helmholtz resonators for the first nineteen overtones of a fundamental having a frequency of 64 vibrations per second; the ten odd-numbered resonators in the series correspond to a fundamental of 128 vibrations per second and its first nine overtones. The resonator consists of a spherical shell of metal or glass; there is a conical protuberance ending in a small aperture, which is to be inserted in the ear; opposite this aperture is an opening, through which the sound waves influence the air in the resonator. The tuning depends upon the volume of air in the resonator and the size of the opening. If one ear is stopped while a resonator is applied to the other, most of the tones existing in the surrounding air will be damped or, in effect, excluded, while if a component sound exists which is of the same pitch as that of the resonator, this particular simple tone affects the ear powerfully.

LECTURE III

METHODS OF RECORDING AND PHOTOGRAPHING SOUND WAVES

The Diaphragm

An adequate investigation of the most interesting characteristic of sound, tone quality, requires consideration of the form of the sound wave; for this purpose it is desirable to have visible records of the sounds from various sources which can be quantitatively examined and preserved for comparative study.

Nearly all the methods which have been developed for recording sound make use of a diaphragm as the sensitive receiver. A *diaphragm* is a thin sheet or plate of elastic material, usually circular in shape, and supported more or less firmly at the circumference. The telephone has a diaphragm of sheet iron; in the talking machine sheets of mica are often used, while the soundboard of a piano is a wooden diaphragm; many other materials may serve for special purposes, such as paper, parchment, animal tissue, rubber, gelatin, soap film, metals, and glass.

Diaphragms respond with remarkable facility to tones of a wide range of pitch and to a great variety of tone combinations. The telephone transmitter, the recording talking machine, and the eardrum illustrate the diaphragm set in vibration by the direct action of air waves; one readily thinks of the diaphragm as being affected by the variations in air pressure which constitute the wave, but it is difficult to realize how the movements can accurately correspond to the composite harmonic motion which represents the particular tone color of a given voice or instrument. However, the reproductions of the telephone and talking machine are convincing evidence that the diaphragm does so respond, at least to the degree of perfection attained by these instruments.

Not only may sound waves cause a diaphragm to vibrate, but what is even more wonderful, a diaphragm vibrating in any manner may set up sound waves in the air; this reverse action of the diaphragm is shown in the receiving telephone, magnetism being the exciting cause, and in the machine which talks, the diaphragm of which is mechanically pulled and pushed by the record. The head of a drum is a diaphragm excited by percussion, the soundboard of a piano is caused to vibrate by the action of the strings, and the vocal chords may be considered as a diaphragm set in vibration by a current of air.

The usefulness of the diaphragm is limited, and sometimes annulled, for both scientific and practical purposes, by certain peculiarities in its action related to what are called its natural periods of vibration; these effects of the diaphragm are considered in Lecture V.

Various instruments employing the diaphragm, which have been useful in research on sound waves, will be described in the succeeding articles.

The Phonautograph

The Scott-Koenig phonautograph, by which sound waves are directly recorded,[33] was perfected in 1859. The instrument consists of a membrane placed at the focus of a parabolic receiver or sound reflector, Fig. 55; a stylus attached to the membrane makes a trace on smoked paper carried

FIG. 55. Koenig's phonautograph for recording sounds.

on a rotating cylinder; a sound produced in front of the receiver causes movements of the membrane which are

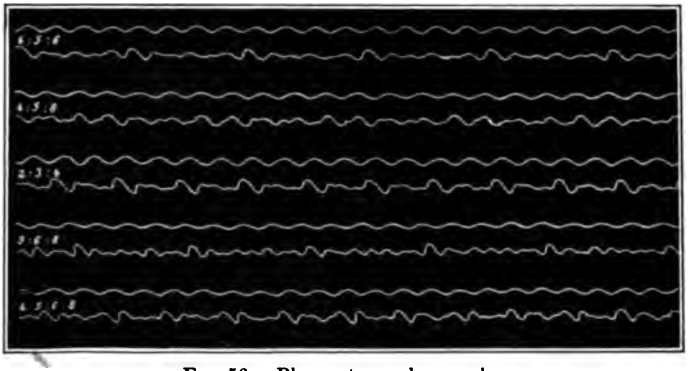


FIG. 56. Phonautograph records.

recorded. A tuning fork with its prongs between the membrane and the paper is mounted on the base of the instrument; a stylus attached to one prong of the fork marks a simple wave line by the side of the trace from the membrane.

Phonautograph records obtained by Koenig are shown in Fig. 56, the lower one of each pair of traces is that of the sound being studied, combinations of organ pipes in this instance, while the upper trace of each pair is from the tuning fork, enabling the determination of the frequencies of the recorded tones. These records are not only small in size, but the essential characteristics are distorted or obliterated by friction and by the momentum of the stylus.

The Manometric Flame

In 1862 Koenig devised the manometric capsule in which the flame of a burning gas jet vibrates in response to the variations in pressure in a sound wave.[34] The capsule *c*, Fig. 57, is divided into two compartments by a partition

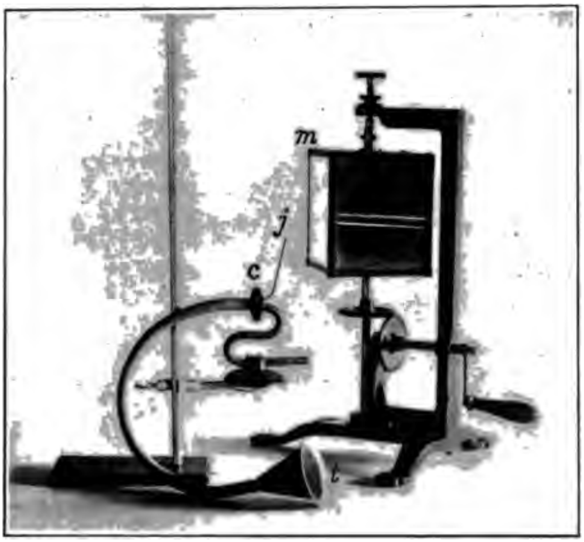


FIG. 57. Koenig's manometric capsule with revolving mirror.

of thin rubber; the variations of air pressure due to the sound wave are communicated through the speaking tube *t* to one side of the partition, while the gas supply for the burning jet *j* is on the other side; the movements of the diaphragm produce changes in the pressure of the gas which cause the height of the flame to vary accordingly.

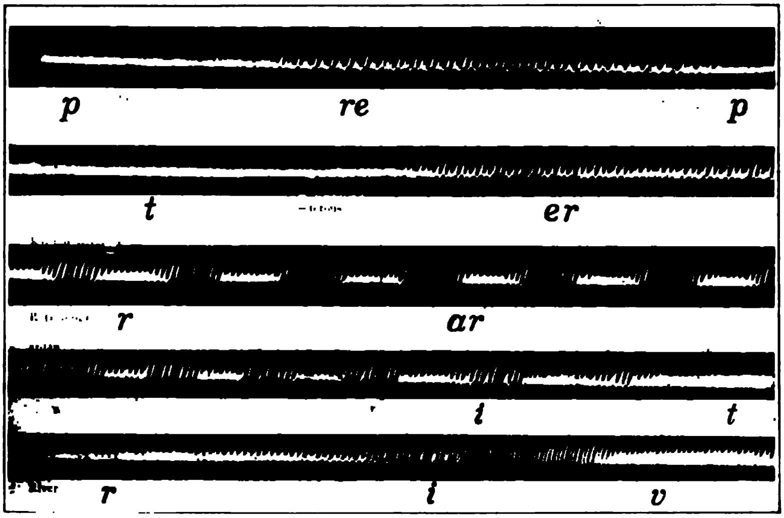


FIG. 58. Manometric flame records of speech by Nichols and Merritt.

The vibrations of the flame may be observed in a revolving mirror *m*.

A method has been devised by Professors Nichols and Merritt in which, by the use of acetylene gas, the flame may be photographed;[35] they have obtained valuable and interesting results in the study of the vowels and spoken words; Fig. 58 shows a portion of a flame record of the vibrations from the spoken words *preposterous* and *Raritan River*. The top line is the syllable *pre*, the second line the syllables *ter-ous;* the next two lines show parts of *rar-i-t*, while the bottom line represents *ri-v*.

The method of recording sound vibrations by photographing the acetylene flame has been still further developed by Professor J. G. Brown, whereby an outline of the wave form is obtained.[36] Fig. 59 shows records made by this method, the boundary between the light and dark portions being the wave form.

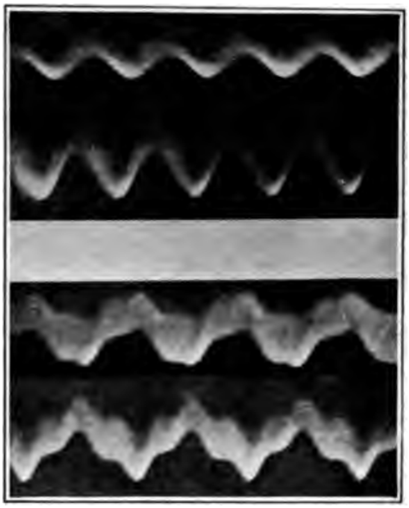


Fig. 59. Vibrating flame records of sounds by Brown.

Fig. 60. Duddell's oscillograph.

THE OSCILLOGRAPH

The telephone invented by Bell in 1876, as well as the microphone transmitter of Hughes (1878), generates electromagnetic waves from the sound waves impressed upon the diaphragm of the transmitter. These waves may be received by the oscillograph, a specialized type of galvanometer, Fig. 60, developed by Blondel (1893) and by Duddell.[37] The electric waves set a minute mirror into corresponding vibrations which may be recorded photographically.] The telephone receiver proves that these vibrations correspond to the sound waves sufficiently to make speech intelligible, but it is known that mechanical and electromagnetic factors produce appreciable alterations of the wave forms.

The method is being continually developed and improved and is of much value, especially in telephone research. Fig.

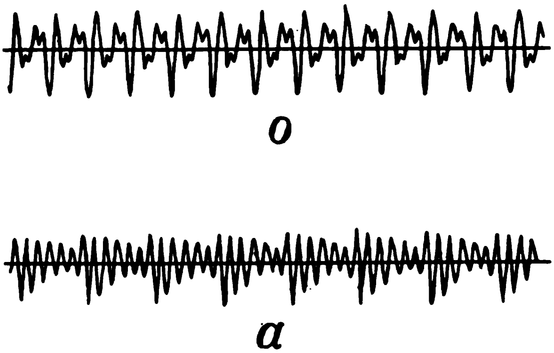


FIG. 61. Records of vowels obtained with the oscillograph.

61 shows a telephone-oscillograph record of the vowel sounds *a* and *o*.[38]

THE PHONOGRAPH

The phonograph, invented by Edison in 1877, originally recorded the movements of a diaphragm by indentations in a sheet of tinfoil supported over a spiral groove in a metal cylinder, Fig. 62. In later machines the movements of the diaphragm are recorded by minute cuttings on the surface of a wax cylinder or disk.

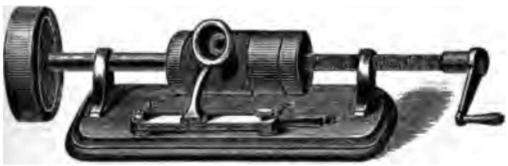


FIG. 62. Phonograph, early form.

A modification of the phonograph was invented by Bell and Tainter and called the graphophone; Berliner introduced the method of etching the original record on a zinc disk, producing the gramophone.

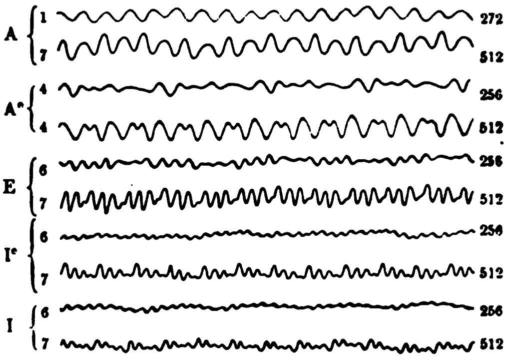


FIG. 63. Vowel curves enlarged from a phonographic record.

Hermann[39] in 1890, and Bevier[40] in 1900, each made photographic copies of phonograph records on an enlarged scale. A delicate tracing point carrying a mirror was so mounted that, as it passed slowly over the record, a beam of light reflected from the mirror fell upon a moving photographic paper or film and registered the wave form. Fig. 63 shows records of vowel sounds obtained by Bevier.

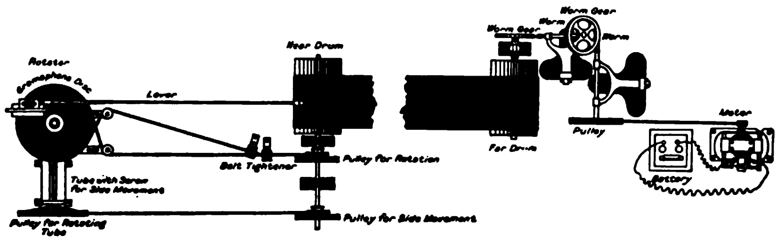


FIG. 64. Scripture's apparatus for tracing talking-machine records.

By means of a tracing apparatus, a top view of which is shown in Fig. 64, Scripture has copied talking machine records enlarged 300 times laterally and about 5 times in length.[41] A disk record is rotated very slowly, one turn in 5 hours, while a tracing point rides smoothly in the groove; by means of a system of delicate compound levers, the lateral movements of the tracer are registered on a moving strip of smoked paper. The whole apparatus is operated by an electric motor, and when started, may be left to continue the tracing to the end, which operation, with the magnification employed, is practicable for a few turns only of the disk. Fig. 65 shows such a tracing from a record of orchestral music, which as here reproduced is magnified about 150 times laterally and 2½ times in length. These copies are probably the best that have been obtained from phonographic records.

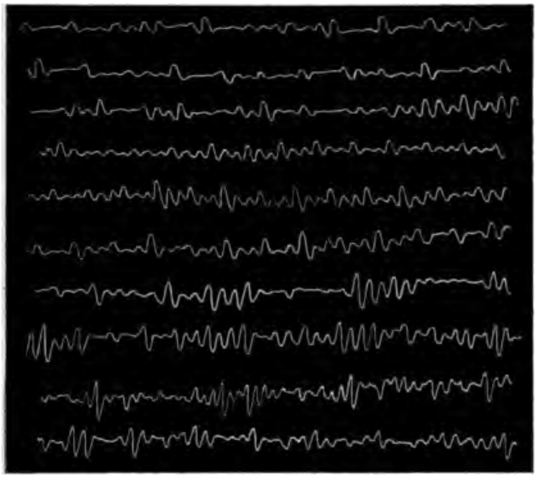


FIG. 65. A tracing, by Scripture, of a record of orchestral music.

Both the process of making the original record in wax and the subsequent enlarging introduce imperfections into the curves; nevertheless these methods have been of great value in many researches in acoustics.

THE PHONODEIK

For the investigation of certain tone qualities referred to in Lecture VI, the author required records of sound waves showing greater detail than had heretofore been obtained. The result of many experiments was the development of an instrument which photographically records sound waves, and which in a modified form may be used to project such waves on a screen for public demonstration; this instrument has been named the "Phonodeik," meaning to show or exhibit sound.[42]

The sensitive receiver of the phonodeik is a diaphragm, *d*, Fig. 66, of thin glass placed at the end of a resonator horn *h*; behind the diaphragm is a minute steel spindle mounted in jeweled bearings, to which is attached a tiny mirror *m*; one part of the spindle is fashioned into a small

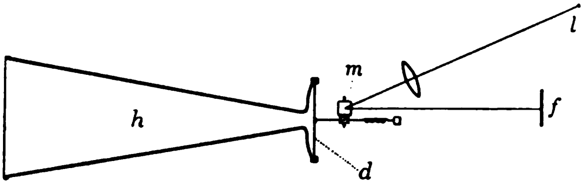


FIG. 66. Principle of the phonodeik.

pulley; a few silk fibers, or a platinum wire 0.0005 inch in diameter, is attached to the center of the diaphragm and being wrapped once around the pulley is fastened to a spring tension piece; light from a pinhole *l* is focused by a lens and reflected by the mirror to a moving film *f* in a special camera. If the diaphragm moves under the action of a sound wave, the mirror is rotated by an amount proportional to the motion, and the spot of light traces the record of the sound wave on the film, in the manner of the pendulum shown in Fig. 11, page 12.

In the instrument made for photography, Fig. 67, the usual displacement of the diaphragm for sounds of ordinary loudness is about half a thousandth of an inch, resulting in

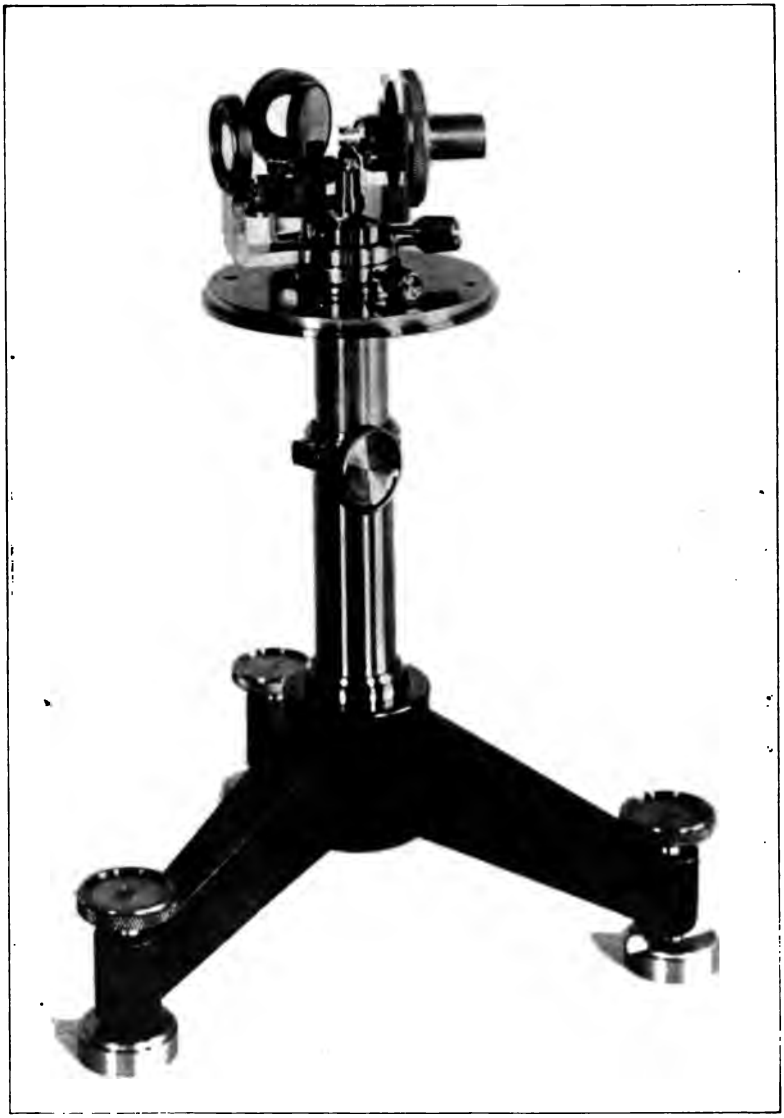


FIG. 67. The phonodeik used for photographing sounds.

an extreme motion of one thousandth of an inch, which is magnified 2500 times on the photograph by the mirror and light ray, giving a record 2½ inches wide; the film commonly employed is 5 inches wide, and the record is sometimes wider than this. The extreme movement of the diaphragm of a thousandth of an inch must include all the small variations of motion corresponding to the fine details of wave form which represent musical quality. Many of the smaller kinks shown in the photographs, such as Figs. 110 and 169, are produced by component motions of the diaphragm of less than one hundred-thousandth of an inch; the phonodeik must faithfully reproduce not only the larger and slower components, but also these minute vibrations which have a frequency of perhaps several thousand per second.

The fulfillment of these requirements necessitates unusual mechanical delicacy; the glass diaphragm is 0.003 inch thick, and is held lightly between soft rubber rings, which must make an air-tight joint with the sound box; the steel staff is designed to have a minimum of inertia, its mass is less than 0.002 gram (less than $\frac{1}{32}$ grain); the small mirror, about 1 millimeter (0.04 inch) square, is held in the axis of rotation; the pivots must fit the jeweled bearings more accurately than those of a watch; there must be no lost motion, as this would produce kinks in the wave, which when magnified would be perceptible in the photograph; there must be no friction in the bearings.

The phonodeik responds to 10,000 complete vibrations (20,000 movements) per second, though in the analytical work so far undertaken it has not been found necessary to investigate frequencies above 5000.

The author wishes to record that the success of the phonodeik in meeting these requirements is due to the friendly interest and exceptional skill of Mr. L. N. Cobb, who constructed the steel staff in accordance with designs which seemed almost impracticable and then mounted it in perfect jeweled bearings.

The camera is arranged for moving films of 5 inches in width and of lengths to 100 feet; there are three separate revolving drums having circumferences of 1, 2, and 5 feet respectively; there is also a pair of drums, each holding 100 feet of film, arranged for winding the film from one to the other during exposure. The single drums are turned by an electric motor, with film speeds varying from 1 to 50 feet per second.

A rheostat for controlling the speed of the motor is placed where it can be reached by the experimenter when he stands near the horn, and there is visible a tachometer which indicates the film speed. For general display pictures a speed of 5 feet per second is convenient, while for records to be analyzed 40 feet per second is suitable; for the latter purpose a short record 1 or 2 feet long, made in $\frac{1}{40}$ or $\frac{1}{20}$ of a second, is sufficient.

The camera is provided with several shutters of various types for hand, foot, and automatic electric release, and for any desired time of exposure; and a commutator on the revolving drum may be used to open and close the shutter at desired points in its revolution.

Besides the record of the wave there are photographed on the film simultaneously a zero line to give the axis of the curve for analysis, and time signals from a stroboscopic fork, $\frac{1}{100}$ second apart, to enable the exact determination of pitch from measurements of the film. The axis and time signals are shown in Fig. 96 and in many others;

when the photograph is intended for display only, these records are sometimes omitted.

For visual observations the camera is provided with a horizontal revolving mirror which reflects the vibrating light spot upward on a ground glass in the form of a wave; an inclined stationary mirror above the ground glass makes the wave visible to the experimenter while the sound is produced. The speed of the revolving mirror and the dimensions in general are so proportioned that the wave appears on the ground glass in the same size and position as when photographically recorded. The speed of the motor may be adjusted till the wave appears satisfactory and the film speed will be automatically varied to correspond; the sound is altered in loudness or quality as desired; when a suitable wave appears on the ground glass, the closing of an electric key or the pressure of the foot on a floor trigger makes the photographic exposure. The photographs are all taken under such conditions that the film moves from right to left, giving the time scale in a positive direction, and that a positive ordinate of the curve corresponds to the compression part of the air wave.

A sound-recording instrument might best be used out of doors, on the roof of a building for instance, to avoid confusion of the records by reflection from the walls; since it is not convenient to work in such a place, the disturbing factors of the laboratory room are minimized by various precautions, such as padding the walls with thick felt; Fig. 68 shows the room in which the photographs are made; the phonodeik with the receiving horn stands on a pier, while the light and moving-film camera are behind the screen. The tuning fork which flashes the time signal is shown at the right.

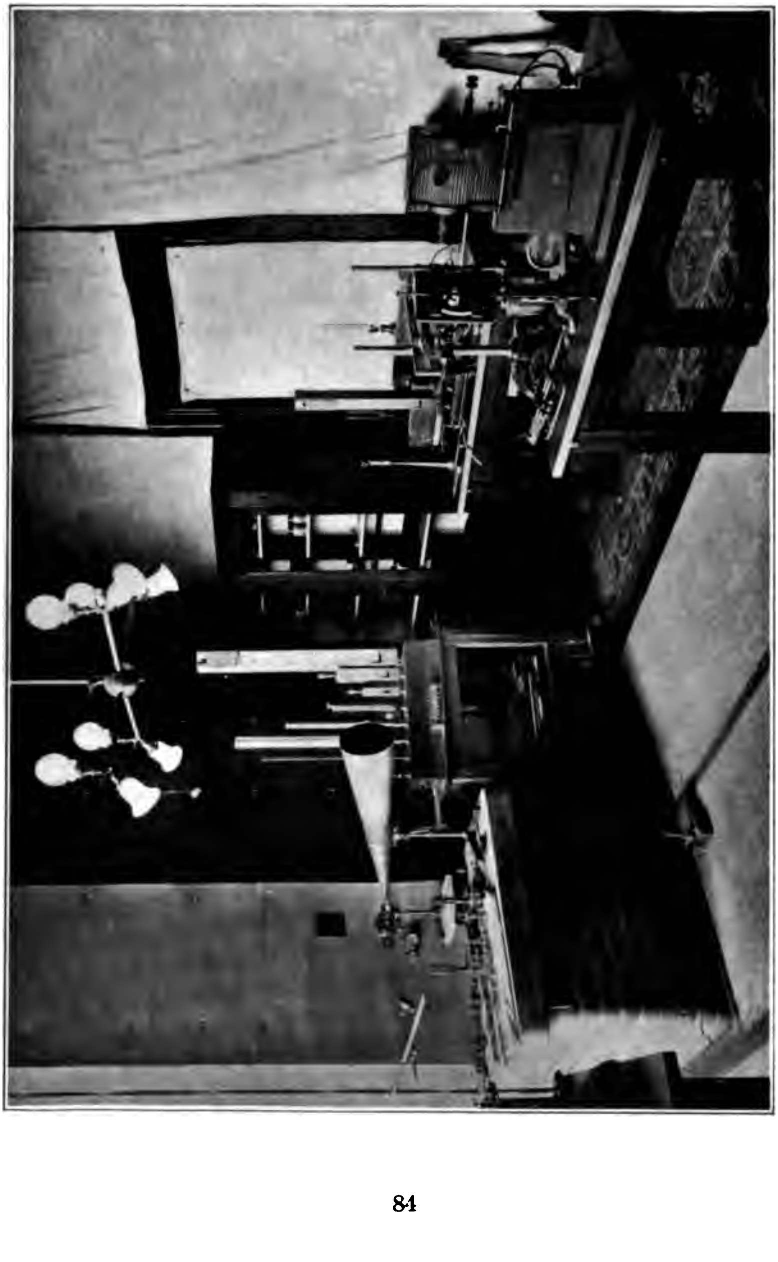


FIG. 68. A laboratory equipped for photographing sound waves.

The Demonstration Phonodeik

The vibrator of the phonodeik employed in research is very minute and delicate, and its small mirror reflects too little light to make the waves visible to a large audience. For purposes of demonstration, a phonodeik has been especially constructed, Fig. 69, which will clearly exhibit the principal features of "living" sound waves.[43] The sound from a voice or an instrument is produced in front of the horn; the movements of the diaphragm with its vibrating mirror cause a vertical line of light which, falling upon a motor-driven revolving mirror, is thrown to the screen in the form of a long wave; the movements of the diaphragm are magnified 40,000 times or more, producing a wave which may be 10 feet wide and 40 feet long.

With this phonodeik a number of experiments may be made in further explanation of the principles of simple harmonic motion and wave forms. When the revolving mirror is kept stationary, the spot of light on the screen moves in a vertical line as the diaphragm vibrates; though these movements are superposed, their extreme complexity is shown since the turning points are made evident by bright spots of light. If the mirror is slowly turned by hand, the production of the harmonic curve by the combination of vibratory and translatory motions is demonstrated. With a tuning fork the simplicity of the sine curve is exhibited; with two tuning forks the combination of sine curves is shown; the imperfect tuning of two forks is demonstrated by a slowly changing wave form; the relations of loudness to amplitude and of pitch to wave length may be illustrated.

The projection phonodeik is especially suitable for exhibit-

FIG. 69. Projection phonodeik for rendering sound waves visible to a large audience.

ing the characteristics of sounds from various sources; as seen on the screen the sound waves are constantly in motion, changing shape and size with the slightest alteration in frequency, loudness, or quality of the source.

(As delivered orally, this Lecture was illustrated with many photographs of sound waves and also by the projection of the sound waves from various sources upon the screen. The greater number of the photographs so used are reproduced in various parts of this book, while the characteristics of the sources of sound are described in Lecture VI.)

Determination of Pitch with the Phonodeik

The photographs obtained with the phonodeik permit a very convenient and accurate determination of pitch; the time signals are given by a standard tuning fork, recording one hundred flashes per second; it is only necessary to compare the wave length and the time intervals to obtain the frequency. Various photographs, as Fig. 96, show the time signals.

A standard clock with a break-circuit attachment may be made to record signals simultaneously with the sound waves; by counting and measuring, the number of waves per second may be determined with precision. When two sounds are being compared by the method of beats, the exact number (including fractions) of beats per second may be determined by photographing the beats together with the time signals.

The phonodeik permits accurate tuning of all the harmonic ratios; if the spot of light is observed without the revolving mirror, its movements take place in a straight line; two tones sounding simultaneously give a composite wave form, the turning points of which are visible as circles of extra brightness on the line, like beads on a string. When the ratio of the component tones is inexact, there is a constant change of wave form which causes the beads to creep along the line; when the ratio is exact, the wave form is constant and the beads are stationary, signifying perfect tuning.

Photographs of Compression Waves

The methods for recording sound so far described show the movements of a diaphragm produced by the varying air pressure of the sound wave. Sound waves consist of alternate condensations and rarefactions which are propagated through space with a velocity of 1132 feet per second; for the tone middle C the distance from one compression to the next is about four feet. It would be very useful indeed if photographs could be obtained of ordinary sound waves in air, but no practicable means has yet been devised for photographing waves of this size.

A method due to Toepler[44] has been successively developed by Mach, Wood, Foley and Souder, and Sabine, by which instantaneous photographs can be obtained of the snapping sound of an electric spark from a Leyden jar. This sound consists of a single wave containing one condensation and one rarefaction, the wave length may be $\frac{1}{16}$ inch or less, and the sound is relatively a loud one, that is, the change in density is considerable. If while such a sound wave is passing over a photographic plate in the dark, the wave is instantaneously illuminated by a single distant electric spark, the light from the spark will be refracted by the sound wave which will then act as a lens and register itself on the plate. There must be one miniature flash of lightning to make the sound, a sort of minute clap of thunder, and a second distant flash a small fraction of a second later to illuminate the thunder wave as it passes outwards. This method is not suitable for recording the peculiarities of ordinary sounds due to various tone qualities, but it is very useful in studying some features of wave propagation.

The most beautiful photographs of this kind have been recently obtained by Professor Sabine and applied by him to the practical problem of auditorium acoustics, to which

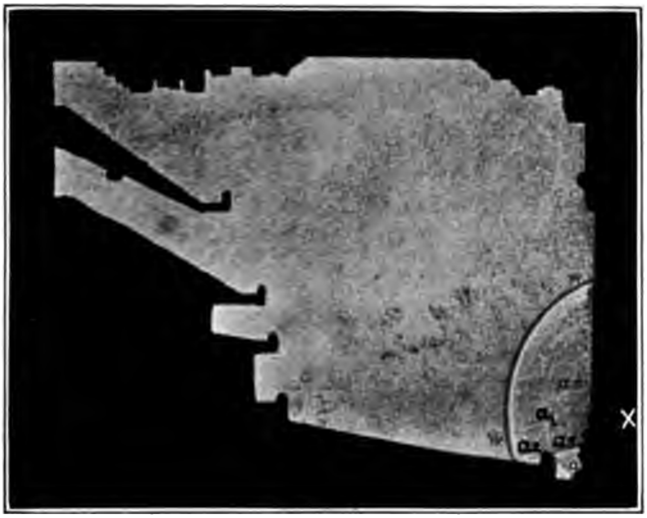


FIG. 70. Cross-sectional model of a theater, with the photograph of a sound wave entering the auditorium.

reference was made in Lecture II. A small cross-sectional model of the auditorium is prepared, as shown in Fig. 70, and the photographic plate is placed behind it; the sound is produced on the stage, at x, and the resulting wave proceeds on its journey into the auditorium, moving at the rate of 1132 feet per second.

The wave length in the experiment is about $\frac{1}{20}$ inch, which is equivalent to a wave length of two feet in the actual audi-

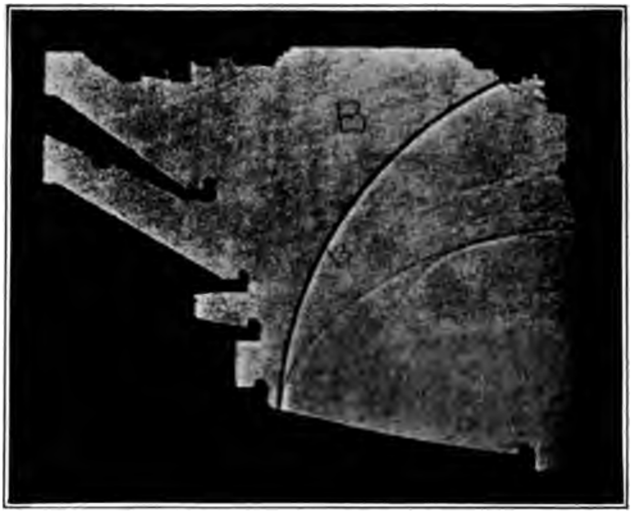


Fig. 71. Position of a sound wave in a theater $\frac{1}{80}$ second after its production on the stage.

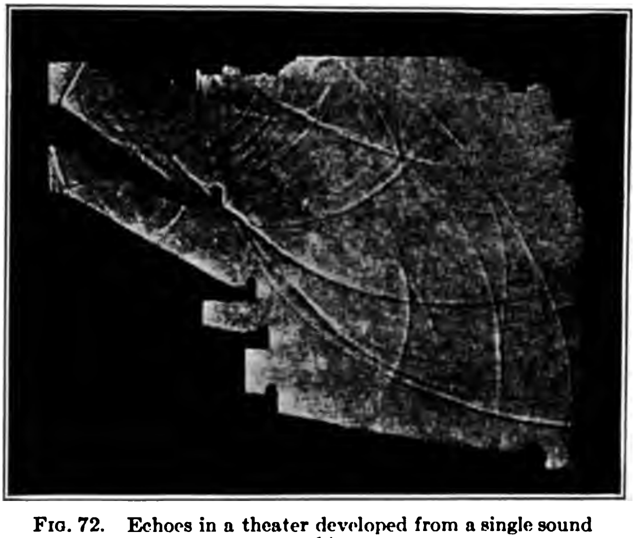


Fig. 72. Echoes in a theater developed from a single sound impulse in $\frac{14}{100}$ second.

torium, corresponding to the musical tone one octave above middle C. The wave in the real auditorium will have reached the position shown in the figure in about $\frac{3}{100}$ second. Various reflected waves or echoes are beginning to appear: a_1 is produced by the screen of the orchestra pit, a_2 is from the main floor, and a_3 is from the orchestra pit floor. Fig. 71 shows the waves about $\frac{5}{100}$ second later, just before the main wave reaches the balcony, and Fig. 72 shows the waves $\frac{14}{100}$ second after the production of the original sound, when the main wave has reached the back of the gallery. The large number of echo waves which seem to come from many directions are actually generated by the one original impulse. The multiple echoes continue to develop with increasing confusion, until the sound is diffused throughout the auditorium, producing the condition called reverberation.

LECTURE IV

ANALYSIS AND SYNTHESIS OF HARMONIC CURVES

Harmonic Analysis

Curves and wave forms such as those obtained with the phonodeik are representative not only of sound, but of many other physical phenomena, and their study is of general importance in science. While inspection and simple measurement will often give some information concerning these curves, as will be explained later, they are in general too complicated for interpretation in their original forms, and several methods of analysis have been developed which greatly assist in our understanding of them.

In the wave method of analysis, often used in optics, the attention is directed to the speed and direction of propagation of the waves in the medium and to their combined effects; in the harmonic method consideration is given primarily to the vibratory character of the movements of the medium, these vibrations being regarded as compounded of a series of motions, which may be infinite in number, but each of which is of a simple definite type.

For the investigation of the complex curves of the sounds of music and speech, the harmonic method of analysis is the most suitable and convenient; it is based upon the important mathematical principle known as *Fourier's Theorem*, the statement and proof of which was first published in Paris, in 1822, by Baron J. B. J. Fourier.[45] For the present purpose Fourier's theorem may be stated as follows: If any curve be given, having a wave length l, the same curve can always be reproduced and in one particular way only, by compounding simple harmonic curves of suitable amplitudes and phases, in general infinite in number, having the same axis, and having wave lengths of l, $\frac{1}{2}l$, $\frac{1}{3}l$, and successive aliquot parts of l; the given curve may have any arbitrary form whatever, including any number of straight portions, provided that the ordinate of the curve is always finite and that the projection on the axis of a point describing the curve moves always in the same direction. Many of the curves studied by this method can be exactly reproduced by compounding a limited number of the simple curves; for sound waves the number of components required is often more than ten, and rarely as many as thirty; in some arbitrary mathematical curves, a finite number of components gives only a more or less approximate representation, while an exact reproduction requires the infinite series of components.

Fourier's theorem may be stated in mathematical form in the *Fourier Equation* as follows: [46]

$$y=\frac{1}{l}\int_0^l y\,dx+\begin{cases}\left[\frac{2}{l}\int_0^l y\sin\frac{2\pi x}{l}dx\right]\sin\frac{2\pi x}{l}+\left[\frac{2}{l}\int_0^l y\sin\frac{4\pi x}{l}dx\right]\sin\frac{4\pi x}{l}+\cdots\\ \left[\frac{2}{l}\int_0^l y\cos\frac{2\pi x}{l}dx\right]\cos\frac{2\pi x}{l}+\left[\frac{2}{l}\int_0^l y\cos\frac{4\pi x}{l}dx\right]\cos\frac{4\pi x}{l}+\cdots\end{cases}\qquad \text{I}$$

In this equation y is the ordinate of the original complex curve at any specified point x on the base line, and l is the fundamental wave length. The principal part of this equation is a trigonometric series of sines and cosines and this (or the whole equation) is often referred to as *Fourier's Series.*

The Fourier equation may be given a simpler appearance by writing it in a second symbolic form:

$$y = a_0 + \begin{cases} a_1 \sin\theta + a_2 \sin 2\theta + a_3 \sin 3\theta + \ldots \\ b_1 \cos\theta + b_2 \cos 2\theta + b_3 \cos 3\theta + \ldots \end{cases} \qquad \text{II}$$

The term a_0 is a constant and is equal to the distance between the chosen base line and the true axis of the curve; if the base line coincides with the axis, $a_0 = 0$, and this term does not appear in the equation of the curve. Since this term has no relation to the shape of the curve, its value is not required in sound analysis; the method for evaluating it, however, is described on page 107.

The other terms of the equation occur in pairs, as $a_1 \sin\theta$, $b_1 \cos\theta$, etc., and each, whether a sine or cosine term, represents a simple harmonic curve. The successive simple curves of the sine series evidently repeat themselves with frequencies of 1, 2, 3, etc., that is, they have wave lengths in the proportions of 1, $\frac{1}{2}$, $\frac{1}{3}$, etc., and the same is true of the cosine series.

Each of the coefficients a_1, b_1, a_2, b_2, etc., is a number or factor indicating how much of the corresponding simple harmonic curve enters into the composite; that is, it shows the amplitude, or height, of the simple wave. For the reproduction of a given curve it may happen that certain of the simple curves are not required, and the corresponding coefficients then have the value zero and their terms do not appear in the Fourier equation of the curve.

A sine and a cosine curve of the same frequency but with independent amplitudes, such as the pairs of curves in the Fourier equation, can be compounded into a single sine (or cosine) curve of like frequency which starts on the axis at a point different from that of the component curves, and which has an amplitude dependent upon the amplitudes of the components. The relation of the starting point of the new curve to that of its components is called its phase, as is explained on page 126. This principle may be stated in symbols as follows, a and b being the amplitudes of the given curves, and A that of the resultant, and P the phase of the new curve:

$$a \sin \theta + b \cos \theta = A \sin (\theta + P),$$

when $$A = \sqrt{a^2 + b^2},$$

and $$\tan P = \frac{b}{a}.$$

If the amplitudes a and b are made the base and altitude, respectively, of a right triangle, Fig. 73, then the hypothenuse is the amplitude A of the resultant curve and the angle which the hypothenuse makes with the base is the phase.

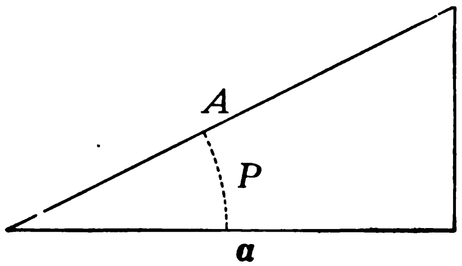


FIG. 73. Amplitude and phase relations of component and resultant simple harmonic motions.

If each pair of sine and cosine terms of the general Fourier equation is reduced in this manner, and if the origin is on the axis of the curve, the equation may be put into the following equivalent form, consisting of a single series of sines:

$$y = A_1 \sin(\theta + P_1) + A_2 \sin(2\theta + P_2) + A_3 \sin(3\theta + P_3) + \ldots \quad \text{III}$$

In this equation A_1 is the amplitude of the first component (the fundamental tone) and P_1 is its phase; while A_2

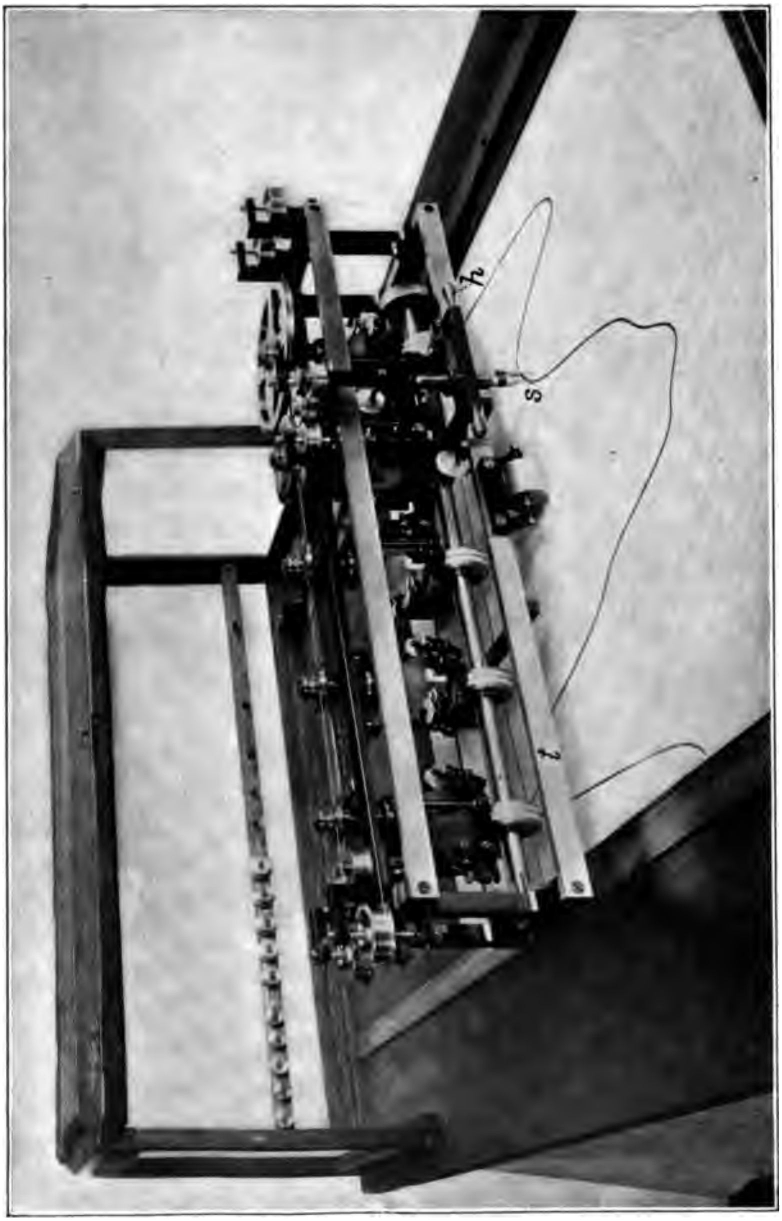


Fig. 74. Henrici's harmonic analyzer.

and P_2 determine the second component (first overtone or octave), etc.

Form III of the Fourier equation is most suitable for representing the results of the physical analysis of a sound, though the actual numerical analysis is obtained in the first form, I, of the equation.

Mechanical Harmonic Analysis

The process of analyzing a curve consists of finding the particular numerical values of the coefficients of the Fourier equation so that it will represent the given curve. Fourier showed how this may be done by calculation (see page 133), but as it is a long and tedious process, requiring perhaps several days' work for a single curve, various mechanical devices have been constructed to lessen the labor.

The coefficients of the various terms, the quantities in square brackets in equation I, have the following form, n being the order of the term:

$$\frac{2}{l}\int_0^l y \sin\frac{2\,n\pi x}{l}\,dx.$$

These are represented by a_1, b_1, etc., in equation II, and are the amplitudes of the component simple harmonic curves. Each definite integral is the area of a certain auxiliary curve on the base l, the nature of which need not be described here; [47] this area, divided by l, gives the mean height of the auxiliary curve, which is then multiplied by 2, giving the amplitude of the corresponding component. There are various area-integrating machines, known in their simple forms as planimeters, which can be adapted to the determination of the areas of a given curve under such conditions as to indicate on the dials the numerical values of the

Fourier coefficients; in some machines the dial readings are the coefficients, in others the dial readings require further slight reduction. Such machines are called *harmonic analyzers*.

Several types of harmonic analyzers are briefly referred to on page 128. The analyzer devised by Professor Henrici, of London, in 1894, based on the rolling sphere integrator, is perhaps the most precise and convenient yet made.[48] An instrument of this type used by the author in the study of sound waves, is shown in Fig. 74, and its operation will be described.

The curve to be analyzed, which must be drawn to a specified scale, as is explained later, is placed underneath the machine; the handles *h* are grasped with the fingers, and the stylus *s* is caused to trace the curve, which requires movements in two directions. The machine as a whole rests on rollers which permit it to be moved to and from the operator, in the direction of the amplitude of the curve, and the stylus is attached to a carriage which rolls along a transverse track *t* in the direction of the length of the curve.

The instrument shown has five integrators; each sphere, made of glass, rests on a roller so that when the curve is traced, the sphere is rotated on a horizontal axis by an amount proportional to the amplitude of the curve; two integrating cylinders with dial indexes rest against each sphere at points 90° apart, Fig. 75, and, by means of a wire and pulley *w* are given rotation about a vertical axis proportional to the movement along the axis of the curve. While each sphere rolls only in amplitude, the cylinders sliding around the sphere take up components of the amplitude motion which are proportional to the sine and cosine of the phase change respectively. The first integrator turns once around its sphere while the tracer moves over one wave length of the fundamental curve, that is, while the stylus is being moved the length of the track t, the next integrator turns twice, and the others three, four, and five times in the same interval. In this manner one

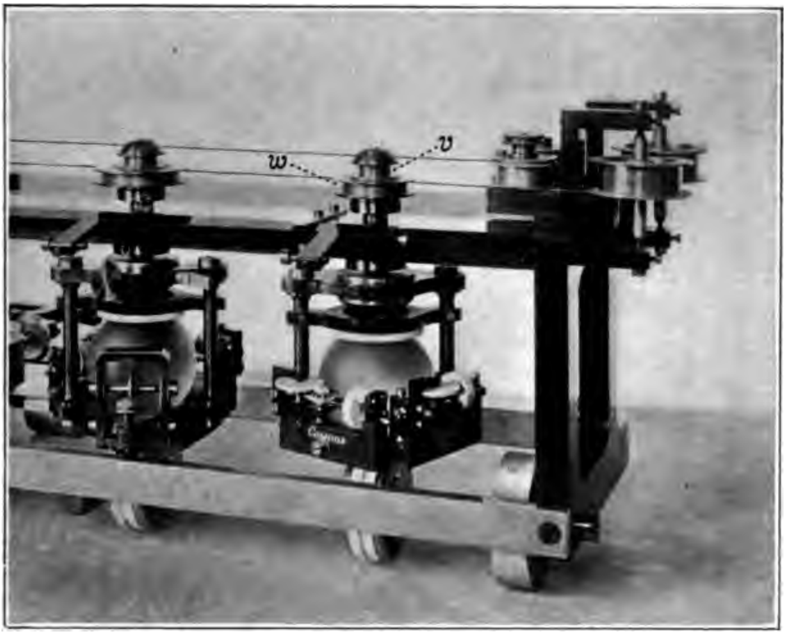


FIG. 75. The rolling-sphere integrator of the harmonic analyzer.

tracing gives the ten coefficients, five sines and five cosines, of the first ten terms of the complete Fourier equation of the curve.

In the Henrici analyzer the sizes of the various parts are so proportioned that the effects of the constant factors of the amplitude terms are mechanically incorporated in the dial readings, which are, without reduction (except for the factor n, mentioned below), the actual amplitudes in millimeters of the components of the curve traced. When the stylus has been moved over one wave length of the fundamental, it must have moved over two wave lengths of the second component, three of the third, and so on; then the integrator for the second component has integrated two waves, and the dial readings are twice the required coefficients; in general, the readings of the nth integrator are n times too large, they are na_n, and nb_n. In the study of sound waves the presence of the factor n is a convenience, for the quantities finally desired are the intensities of the

FIG. 76. Photograph of the sound from a violin.

components, and, as explained on page 167, the loudness of any component is proportional to $(nA_n)^2$.

By changing the wire to the smaller pulleys v on the integrators, the spheres are turned six, seven, eight, nine, and ten times while tracing the wave, and the dials indicate the sine and cosine coefficients for the components from six to ten.

By a reconstruction of the analyzer (in 1910) which it was necessary to carry out in our own instrument shop, the operation of the instrument has been extended from ten to thirty components with precision, six tracings being required for the larger number.[49]

The analysis of the sound wave from the tone $B_4 = 995$, played on the E string of a violin, will be considered. This curve, which is shown in Fig. 76, is comparatively simple. When the curve is analyzed with the machine, the operation proceeds in accordance with the method shown in the Fourier equation I, but the mechanical integrators give the result in form II, and the actual equation read from the dials is as follows:

$$y = 151 \sin \theta - 67 \cos \theta + 24 \sin 2\theta + 55 \cos 2\theta$$
$$+ 27 \sin 3\theta + 5 \cos 3\theta.$$

The analyzer, which has five integrators, gives at the same time with the above the coefficients of the terms involving

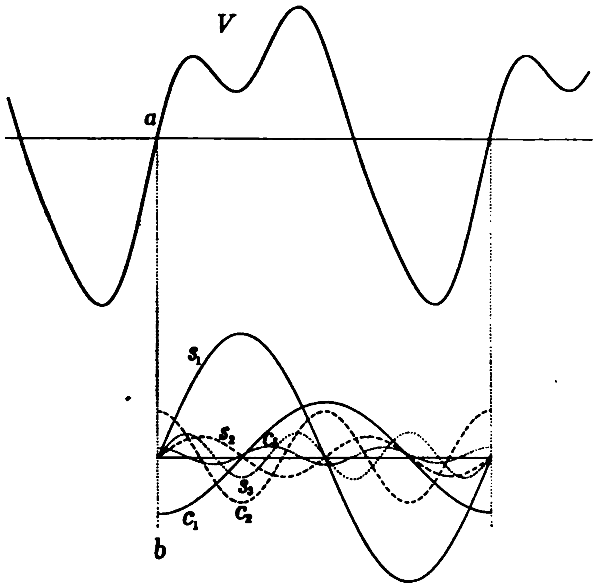


FIG. 77. Curve of a violin tone and its sine and cosine components.

4θ and 5θ; in this instance the latter coefficients are very small, and for simplicity they are omitted. In other words the analysis shows this curve to be composed of three components only, each of which is represented by a pair of sine and cosine terms.

In practical work, each pair of sine and cosine terms is at once reduced to a single term, but for the sake of illustration the graphic interpretation of the equation in its present form is given in Fig. 77; there are six simple curves, a sine and a cosine curve for each of the three frequencies, all starting from the same initial line *ab*; the sine curves are indicated by s_1, s_2, and s_3, and the cosines by c_1, c_2, and c_3. These six curves added together, or made into a composite, will accurately reproduce the violin curve V.

As explained on page 95, each pair of these curves can be reduced to a single equivalent curve, and the six components thus become three. The reduction for the first pair of terms gives the equation:

$$151 \sin \theta - 67 \cos \theta = 165 \sin (\theta + 336°).$$

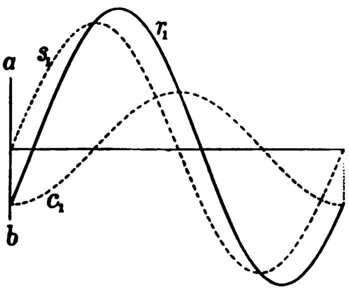


FIG. 78. The resultant of sine and cosine curves.

The graphic interpretation of this reduction is shown in Fig. 78, in which r_1 is the resultant of s_1 and c_1 and is the true representation of the fundamental of the violin curve. The second and third pairs of curves are similarly reduced, giving the curves for the first and second overtones, and the final Fourier equation for the violin curve, in form III, is:

$$y = 165 \sin (\theta + 336°) + 60 \sin (2\,\theta + 66°) + 27 \sin (3\,\theta + 11°).$$

This is the form of equation usually desired in physical investigations; its graphical interpretation is shown in

Fig. 79, which shows the original violin curve at the top, with its three true components, representing partial tones, drawn separately to show the amplitudes and phases (starting points) more distinctly.

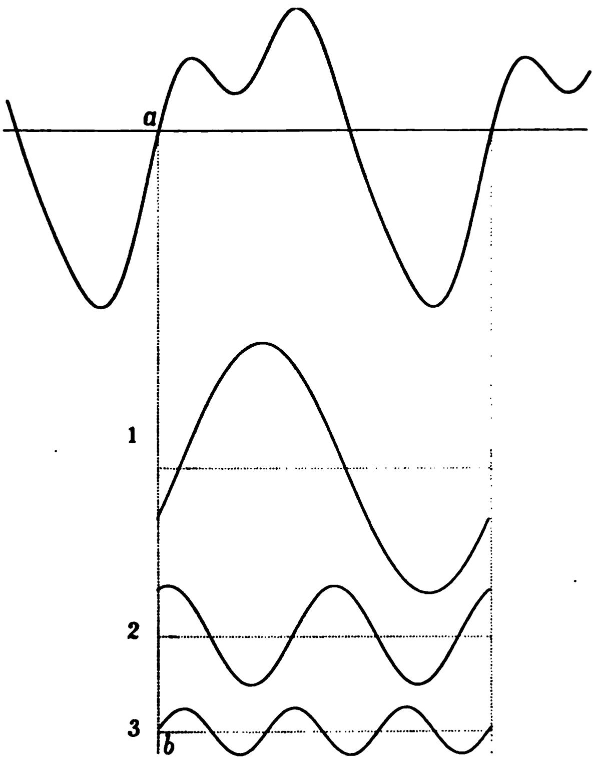


FIG. 79. Curve of a violin tone and its three harmonic components.

The complete analysis and synthesis of a more complicated curve is described later in this Lecture.

AMPLITUDE AND PHASE CALCULATOR

The reduction of the double Fourier series consisting of sines and cosines to the single series of sines with differing phases is usually carried out by numerical calculation, as has been indicated. The need for a more expeditious method, where a large number of curves are being analyzed, has resulted in the design and construction in our own laboratory of a machine, Fig. 80, which accomplishes the purpose in a satisfactory manner.[49] This amplitude-and-

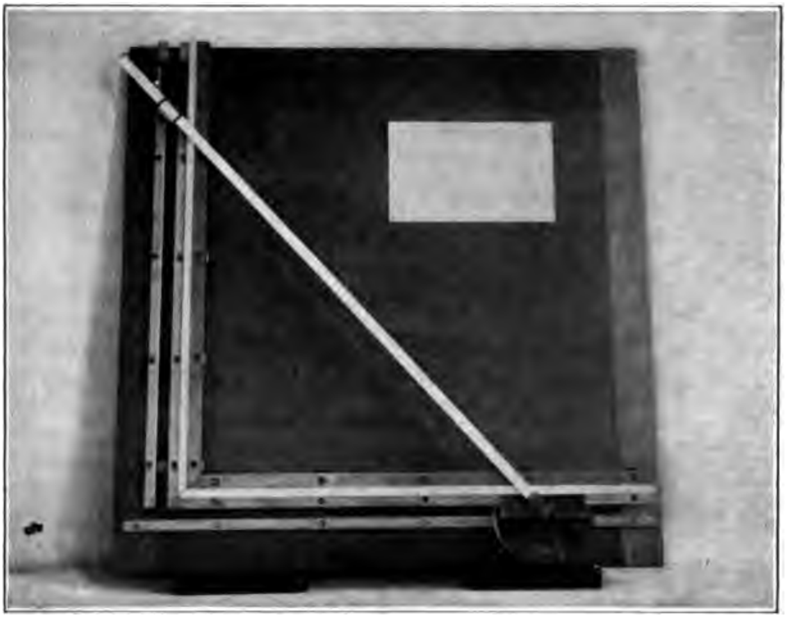


Fig. 80. Machine for calculating amplitudes and phases in harmonic analysis.

phase calculator is essentially a machine for solving right triangles.

The machine has two grooves at right angles to each other, provided with linear graduations; in the grooves are movable sliders which carry the graduated hypothenuse bar; one end of the hypothenuse is attached to a special angle measurer, while the other end slides through a support which also bears an index for reading the length of the bar.

The pair of coefficients a and b of the general Fourier equation, which are given by the analyzer, are set off as the base and altitude, respectively, of the triangle, when the length of the hypothenuse is the amplitude A of the resultant.

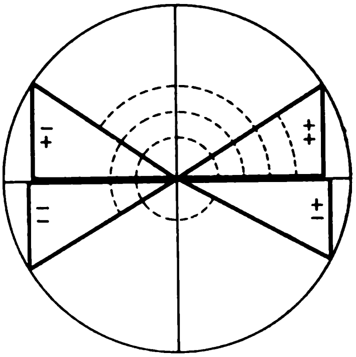


FIG. 81. Phase angles in four quadrants.

The phase of the resultant curve, which is determined by the equation,

$$\tan P = \frac{b}{a},$$

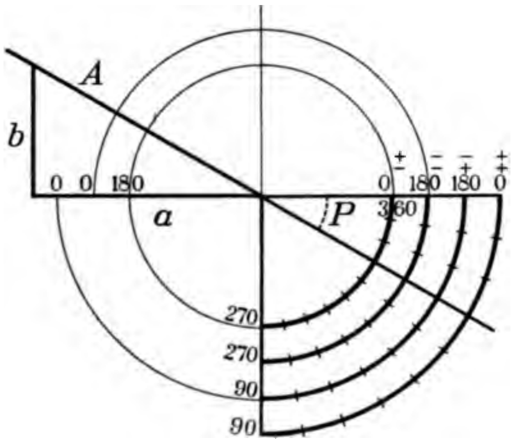


FIG. 82. Scheme for measuring phases in one quadrant.

may have any value from 0° to 360°, since a and b may have either the positive or the negative sign. For the same numerical values of a and b, and therefore for the same value of A, there may be four different values of the phase angle; as indicated in Fig. 81, for $+a$, $+b$, the angle will have a value between 0° and 90°; for $-a$, $+b$, the angle has a value between 90° and 180°; for $-a$, $-b$, it lies between 180° and 270°, and for $+a$, $-b$, it is between 270° and 360°. A special angle measurer with four graduations of a quadrant each might be used for the four possible combinations of algebraic signs; Fig. 82 illustrates a scheme for such graduations.

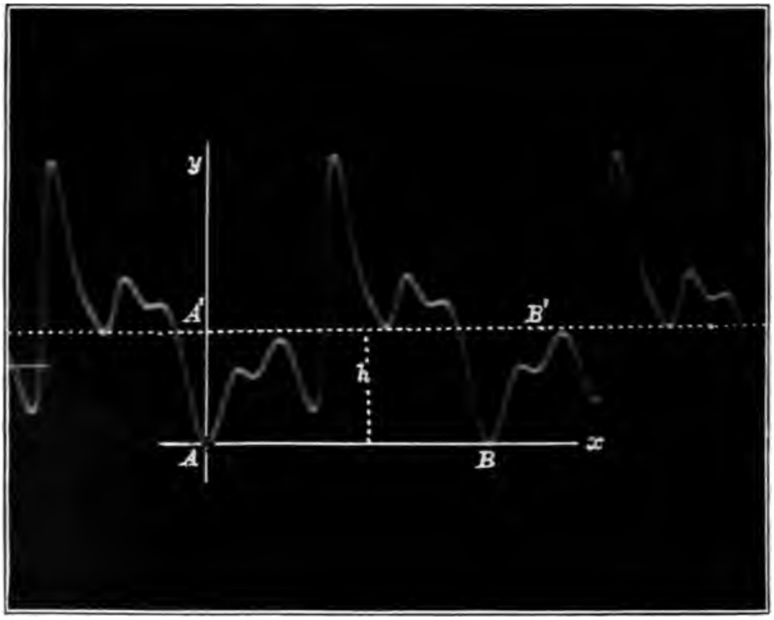


FIG. 83. Finding the axis of a curve.

As a single graduated arc may be provided with two sets of numbers, one on either side, two quadrant graduations are sufficient. To prevent confusion a movable cover is provided for the graduations; this has four apertures so shaped that any one, and one only, of the four sets of numbers is visible at one time, according to the position of a spring catch attached to the cover. There are four positions for this catch marked with the possible combinations of algebraic signs of a and b; when the cover is set, one reads the true phase angle without any reduction.

Axis of a Curve

If the axis of a curve is unknown and is required, it is necessary to determine the first or constant term of the

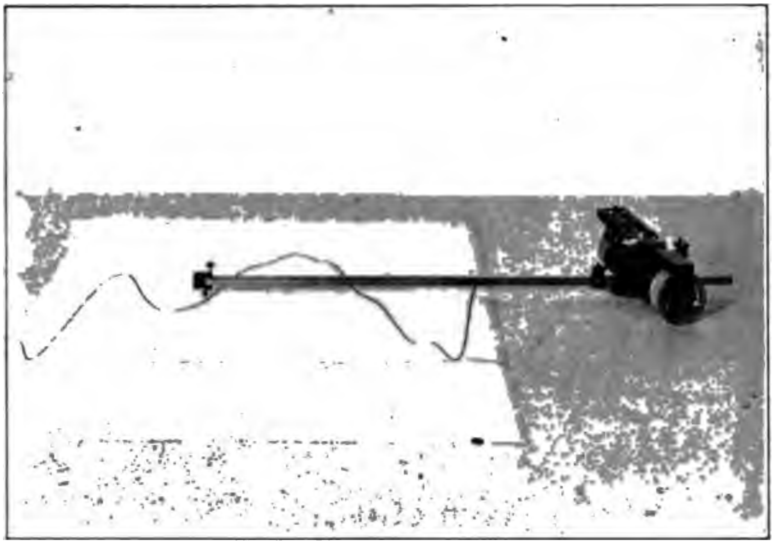


Fig. 84. Finding the true axis of a curve with the planimeter.

Fourier equation, forms I and II; this term consists of the area included between the arbitrarily assumed base line and the curve, divided by the base, and therefore it represents the mean height of the curve from the base. A line drawn through the curve parallel to the base and distant from it by the mean height of the curve, will be the true axis of the curve, since that part of the area between the curve and this axis which is above the axis must be equal to that which is below the axis.

Let it be required to find the axis of the curve shown in Fig. 83. When no base line is given, any line parallel to the axis may be used, such as a line touching the crests or troughs of two waves, *AB*, or any other line through points on two waves which are in the same phase. The area between the assumed base and the curve is measured with a planimeter of any type; this area divided by the wave length is the distance *h* from the base *AB* to the true axis *A'B'*. Fig. 84 shows a precision planimeter with a rolling sphere integrator, in position for the axis determination of a curve, that is, for finding the first or constant term of the Fourier equation.

The constant term gives no information regarding the nature or shape of the curve, it merely gives its position with regard to the base line incidentally employed in drawing or tracing the curve. Ordinarily this term is not required in sound analysis.

Enlarging the Curves

For use with the Henrici analyzer it is necessary that the wave length of the curve which is traced shall be such that when the tracing point moves along its guiding tracks a distance equal to the wave length, the integrator for the first term shall make exactly one revolution around the rolling sphere; in the instrument illustrated the wave length must be 400 millimeters, about 16 inches.

The photographs of sound waves obtained with the phonodeik have wave lengths varying from 25 to 100 millimeters; these waves are enlarged with the apparatus shown in Fig. 85. The photographic film negative of the wave is placed in an adjustable holder *f*, on an optical-bench projection lantern, the curve being projected on a movable easel *e*; adjustments are made until the projected wave is of the proper size, is well defined, and has its axis horizontal; the curve is then traced with a pencil on a sheet of paper. The initial point is chosen merely with reference to convenience in determining the length of one wave, as *a*, Fig. 96, page 122, where the curve crosses the axis. The time required for the operation of enlarging a curve is less than five minutes.

Thus all curves as analyzed are of the same wave length,

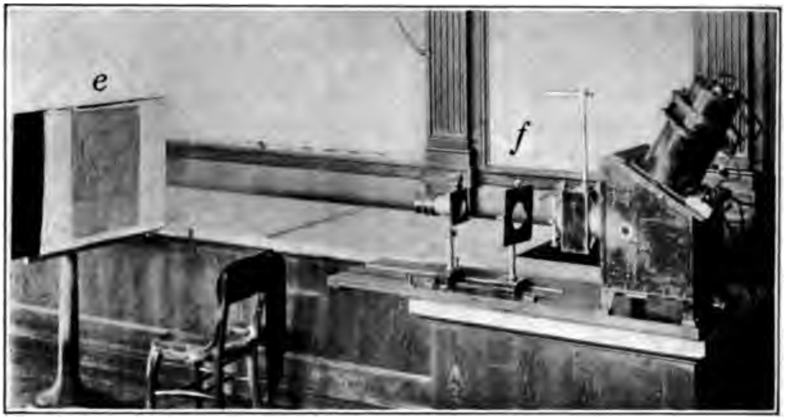


FIG. 85. Apparatus for enlarging curves by projection.

regardless of their original size and frequency, and as they are drawn on a standard sheet of paper, 19 by 24 inches, filing is facilitated. The harmonic synthesizer, described later, draws curves of this same wave length, 400 millimeters, which permits a direct comparison of the analyzed and synthesized curves.

Some of the simpler analyzers mentioned later may be used with a curve of any size such as the original photograph, but the results read from the machine require further reduction for each individual curve and component; as already stated, with the Henrici analyzer, the machine readings are final, requiring no reduction, so far as analysis is concerned. Where many curves are being exhaustively studied by analysis and synthesis, the enlargement to standard wave length is not a disadvantage.

Synthesis of Harmonic Curves

It is often required to perform the converse of the analytical process which has been described, that is, to recombine several simple curves to find their resultant or composite curve; this is *harmonic synthesis.* The synthesis of curves can be accomplished by calculation in some instances, and always by graphic methods by adding the measured ordinates of the component curves and plotting the results; since both of these methods are laborious, machines called *harmonic synthesizers* have been designed to facilitate the work.

A harmonic synthesizer is a machine which will generate separate simple harmonic motions of various specified frequencies, amplitudes, and phases, and will combine these into one composite motion which is recorded graphically.

One of the earliest synthesizers was made in London about 1876 by Lord Kelvin (see page 129), to be used as a tide-predicting machine; it is based upon the pin-and-slot device described in Lecture I. A cord fixed at one end, Fig. 86, passes around several pulleys and at the other end is attached to a pencil, which makes a trace on a moving chart. The pulley *a* is attached to a pin-and-slot device, which moves up and down with a simple harmonic motion; the cord will transmit this motion to the pencil, doubled in amount; if the chart moves continuously, the trace is a simple harmonic curve of a frequency depending upon the rapidity with which the crank-pin is rotated, and of an amplitude depending on the distance of the pin from the center of its crank; the wave length of the curve depends upon the speed with which the chart moves. If another pulley *b* is attached to a second pin-and-slot device rotating twice as fast as the first, it will give the pencil a simple harmonic motion of twice the frequency of the first.

It is evident from the manner in which the cord passes

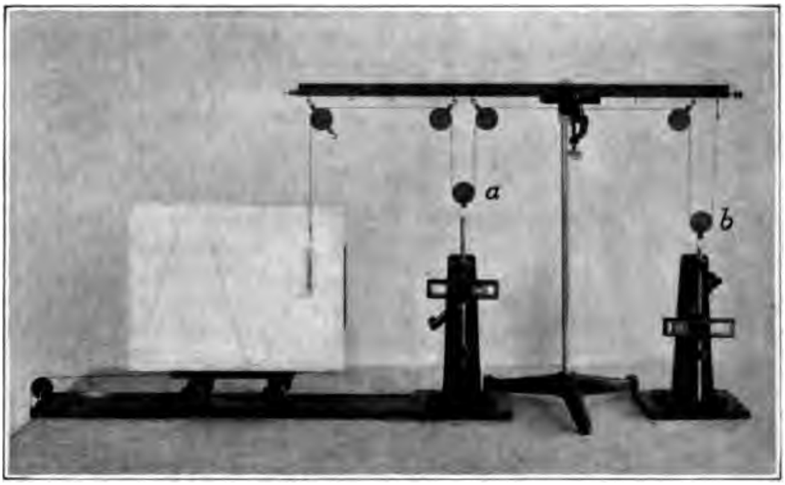


Fig. 86. Apparatus illustrating the method of harmonic synthesis.

around the system of pulleys that if the two devices operate simultaneously, the pencil will have a composite motion which is the sum of the two components, and the trace will be the synthetic curve. The scheme may be extended to include any number of simple harmonic motions of any desired frequencies, phases, and amplitudes.

Two harmonic synthesizers, especially for the study of sound waves, have been designed and constructed in the laboratory of Case School of Applied Science, one having

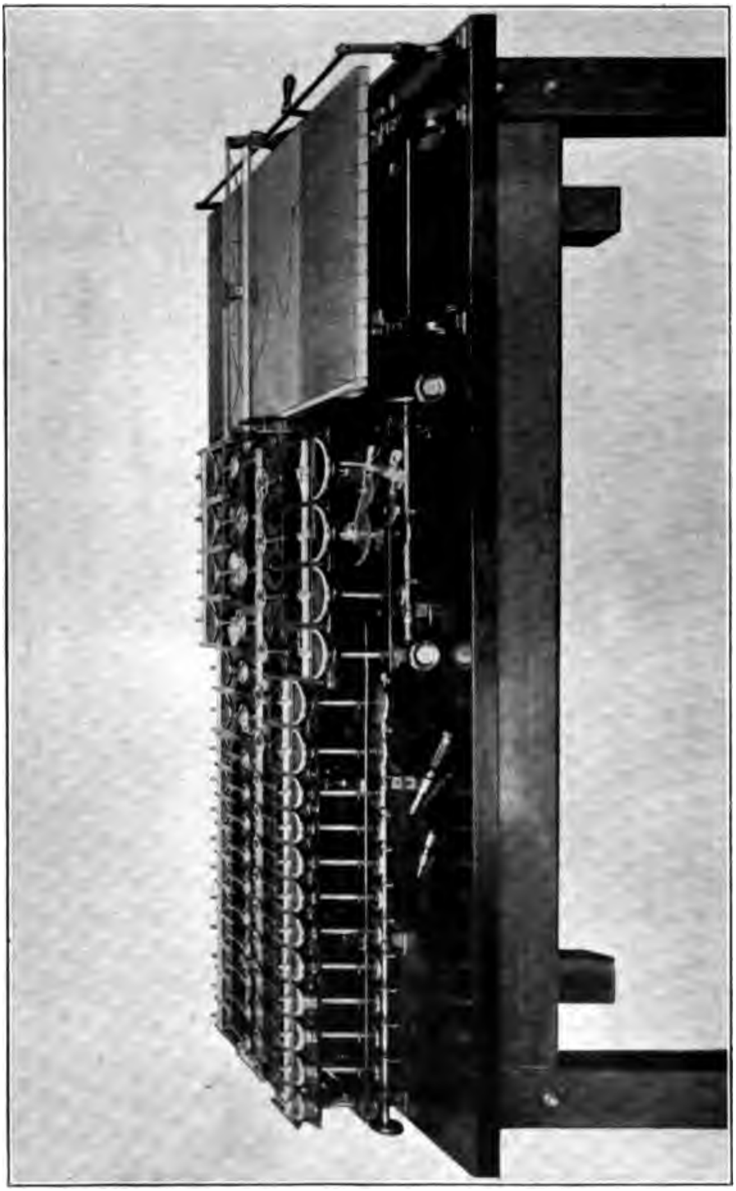


Fig. 87. Harmonic synthesizer with thirty-two elements.

ten components and the other thirty-two. The ten-component machine, finished in 1910, was soon found inadequate for the study of musical sounds and was dismounted in 1914, upon the completion of the thirty-two-component synthesizer shown in Fig. 87. The latter machine is perhaps more convenient for the study of harmonic curves in general than any other which has been constructed; it draws curves with great accuracy and on a very large scale; the drawing board is 24 by 34 inches in size, but by shifting the paper and pen a curve of almost any size may be drawn. The largest single component curve may be 28 inches wide and have a wave length of 32 inches; the highest component may be 4 inches wide. The wave length commonly used is 400 millimeters, about 16 inches, the same as that used with the analyzer, but larger or smaller wave lengths are easily arranged. Thirty of the elements are provided with gears giving the relative frequencies 1, 2, 3 . . . 30; the other two elements are arranged with change-gears, like a lathe head, which permits their easy setting for higher or lower frequencies or for inharmonic frequencies. The machine can be quickly set to give the frequencies 1, 2, 4, 6, and all even terms to 60; or for the series 1, 3, 6, 9, and all multiples of 3 to 90. The mechanical arrangements permit the amplitude and phase of any component to be readily set to any value; all the graduated circles and scales are on the upper surface of the machine and are of white celluloid. There are special scales showing the phase and amplitude of the synthesized curve. All the motions which affect the separate components as they are being compounded and synthetically drawn are provided with ball bearings, to eliminate friction and lost motion; the motion is so accurately transmitted to the pen that a wave can be clearly drawn in which the amplitude is less than 0.2 millimeter (less than $\frac{1}{125}$ inch). This machine is described in detail, with specimen curves, in the Journal of the Franklin Institute.[49]

A ten-component curve can be synthesized in about five minutes, while the machine may be set for thirty components and the curve drawn in twelve minutes.

In the study of sound waves the synthesizer is chiefly used to verify the correctness and sufficiency of the analyses. The several unit devices of the machine having been set to reproduce the separate components in exact sizes and phases, the tracer will draw the resultant curve. If this resultant curve is exactly like the original which was analyzed, the analysis is correct and complete, and the fact is recorded by tracing the synthetic curve over the original in a contrasting color of ink. Fig. 99, page 127, shows the synthetic reproduction of the analysis of an organ-pipe curve (Figs. 96 and 98) drawn on the photograph itself.

The synthesizer is also useful for drawing a curve corresponding to the average of several photographed curves, and for drawing curves of any assumed composition, as in trial analysis. After a photographed curve has been analyzed and the components have been corrected for instrumental disturbances, as explained in Lecture V, it is often useful to draw the corrected synthetic curve, as is illustrated in Fig. 132, page 173.

The synthesizer is useful in preparing illustrations, such as many of those required in these lectures; it would be very difficult to draw the curves of correct form by any other means.

When the synthesizer is set for any curve, if the handle is turned till the phase circle for the first component reads

0°, the circles for the other components show, without calculation, the relative phases, or, as sometimes called, the epochs, of the several components; the tracing point will now be at what may be considered the initial point of the wave, though in general this will not be where the curve crosses the axis. These relations are further explained in connection with the analysis of the curve shown in Fig. 96, on page 122.

The mathematician finds the harmonic synthesizer useful for the investigation of many kinds of curves; the properties of periodic functions and the convergency of series can be shown graphically. The equation of the wave form made up of two straight lines, as shown in Fig. 88, is represented by the infinite series

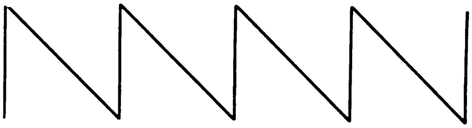


FIG. 88. A geometrical form.

$$y = 2 [\sin x + \tfrac{1}{2} \sin 2 x + \tfrac{1}{3} \sin 3 x + \tfrac{1}{4} \sin 4 x + \ldots],$$

the wave length being equal to $2 \pi = 6.28^{+}$.

The manner in which such an angular geometrical figure may be built up from smooth curves is shown by drawing curves representing different numbers of terms of the series. The first term only, $y = 2 \sin x$, is represented in Fig. 89, *a*; *b*, *c*, *d*, *e*, and *f* represent the curves obtained when two, three, four, five, and ten terms, respectively, are used. Fig. 90 is the curve obtained when thirty terms are included. These curves are graphic illustrations of the convergence of this series; the more terms employed, the closer the result approximates the given form; an infinite number of terms would be required to reproduce the figure exactly.

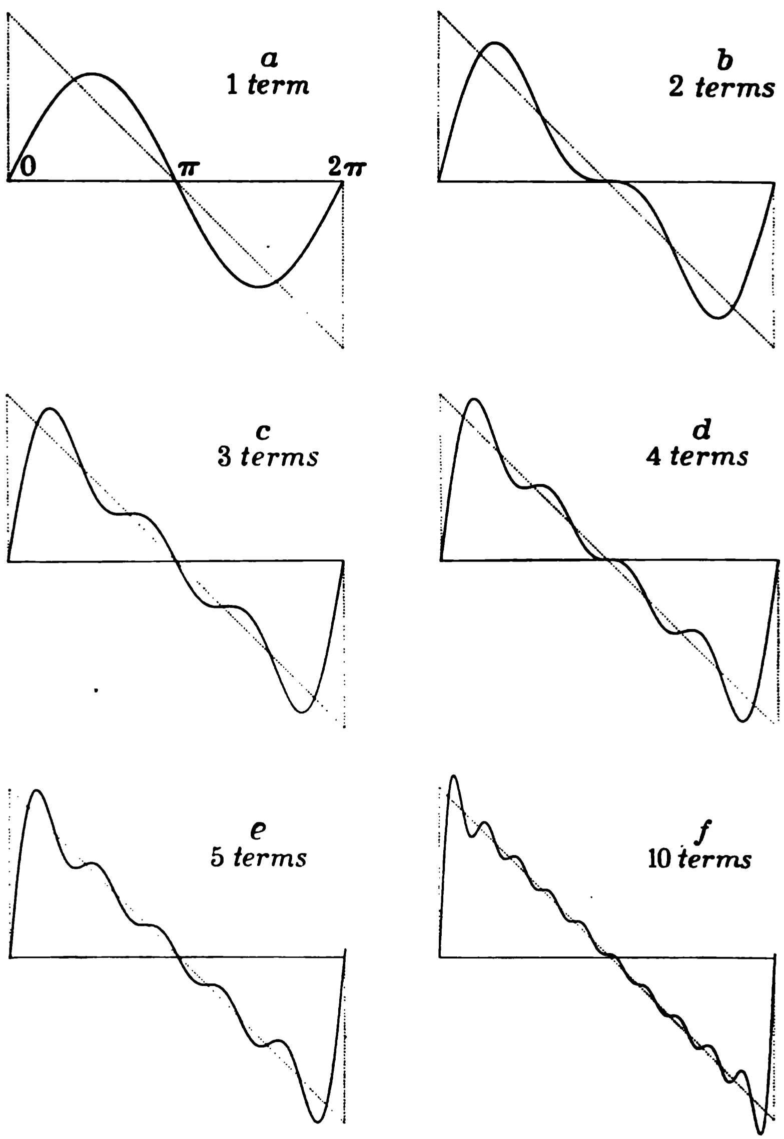


FIG. 89. Forms obtained by compounding 1, 2, 3, 4, 5, and 10 terms of the series $y = 2[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \ldots]$.

Fig. 91 is a curve made up of the same components as enter into the curve shown in Fig. 90; the only difference is

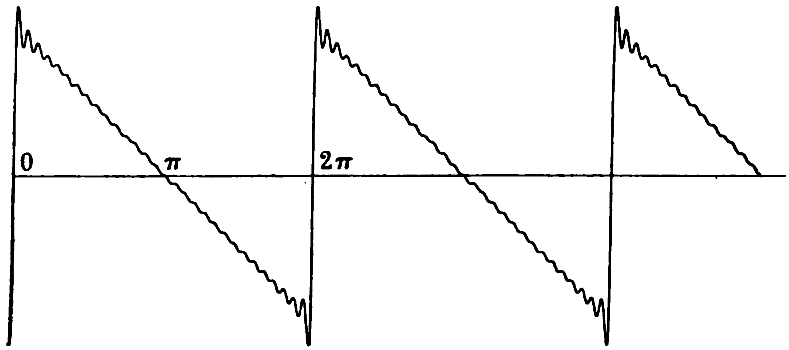


FIG. 90. Curve obtained by compounding 30 terms of the series $y = 2\,[\sin x + \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x + \ldots]$.

that the phase of each component has been changed by 90°; that is, the sines become cosines.

A further interesting variation is obtained by using the

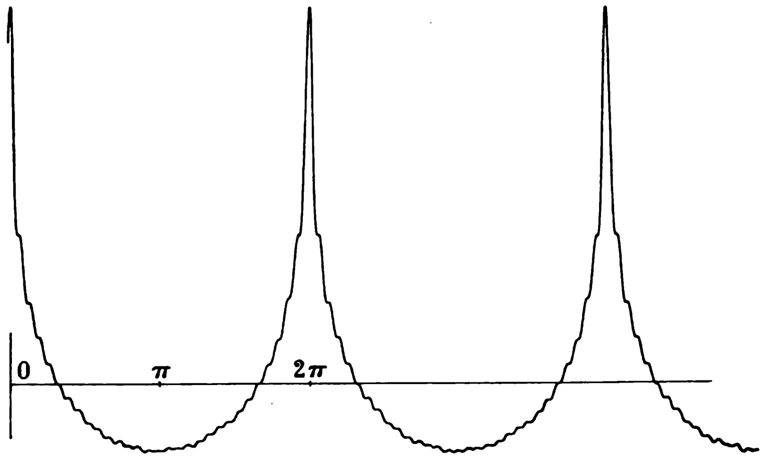


FIG. 91. Curve obtained by compounding 30 terms of the series $y = 2\,[\sin (x + 90°) + \frac{1}{2} \sin (2x + 90°) + \frac{1}{3} \sin (3x + 90°) + \ldots]$, which is equivalent to $y = 2\,[\cos x + \frac{1}{2} \cos 2x + \frac{1}{3} \cos 3x + \ldots]$.

odd-numbered terms only of the first series, producing the form shown in Fig. 92.

If the phases of the alternate terms of the odd-term series

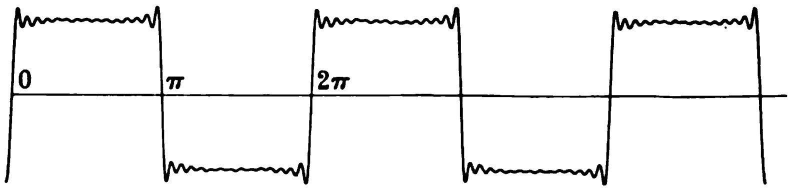


FIG. 92. Curve obtained by compounding 15 terms of the series $y = 2\,[\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots]$.

are changed by 180°, the curved form shown in Fig. 93 is obtained.

The arbitrary nature of the curves that may be studied

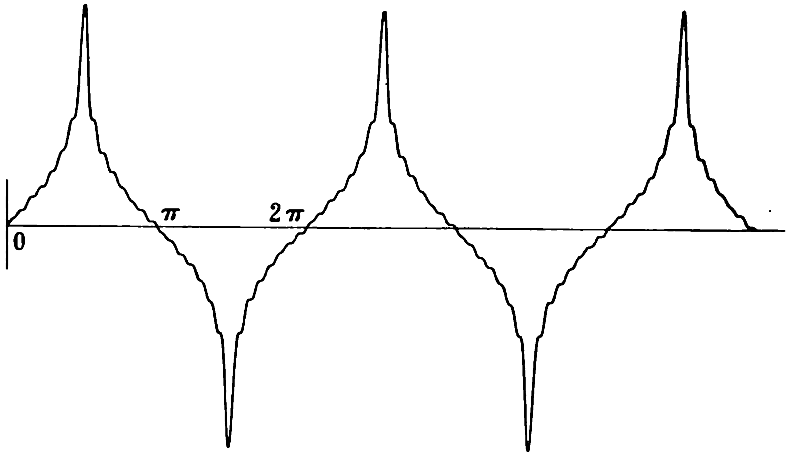


FIG. 93. Curve obtained by compounding 15 terms of the series $y = 2\,[\sin x + \frac{1}{3}\sin (3x + 180°) + \frac{1}{5}\sin 5x + \frac{1}{7}\sin (7x + 180°) + \ldots]$, or $y = 2\,[\sin x - \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x - \frac{1}{7}\sin 7x + \ldots]$.

by the Fourier method is further illustrated by the analysis and synthesis of a portrait profile. The original portrait is shown in the center of Fig. 94, while a tracing of the profile is given at the left, *O*. The curve was analyzed to thirty terms, but the coefficients of the terms above the eighteenth were negligibly small. The equation of the

O *S*

FIG. 94. Reproduction of a portrait profile by harmonic analysis and synthesis.

curve is as follows, the numerical values corresponding to a wave length of 400:

$$
\begin{aligned}
y = \ & 49.6 \sin (\theta + 302^\circ) + 17.4 \sin (2\theta + 298^\circ) \\
& + 13.8 \sin (3\theta + 195^\circ) + 7.1 \sin (4\theta + 215^\circ) \\
& + 4.5 \sin (5\theta + 80^\circ) + 0.6 \sin (6\theta + 171^\circ) \\
& + 2.7 \sin (7\theta + 34^\circ) + 0.6 \sin (8\theta + 242^\circ) \\
& + 1.6 \sin (9\theta + 331^\circ) + 1.3 \sin (10\theta + 208^\circ) \\
& + 0.3 \sin (11\theta + 89^\circ) + 0.5 \sin (12\theta + 229^\circ) \\
& + 0.7 \sin (13\theta + 103^\circ) + 0.3 \sin (14\theta + 305^\circ) \\
& + 0.4 \sin (15\theta + 169^\circ) + 0.5 \sin (16\theta + 230^\circ) \\
& + 0.5 \sin (17\theta + 207^\circ) + 0.4 \sin (18\theta + 64^\circ).
\end{aligned}
$$

This equation was set up on the synthesizer, and the portrait, as drawn by the machine, is shown at the right, *S*, Fig. 94.

If mentality, beauty, and other characteristics can be considered as represented in a profile portrait, then it may be said that they are also expressed in the equation of the profile.

Since the profile is reproducible by compounding a number of simple curves, it is possible to compound the simple tones represented by these curves in such a way that the resulting wave motion of the combined sounds shall be the periodic repetition of the profile. Fig. 95 is a drawing of such a wave. The reproduction of vowel wave forms, shown

FIG. 95. Wave form obtained by repeating a portrait profile.

in Fig. 181, page 250, is a similar synthetic experiment. In this sense beauty of form may be likened to beauty of tone color, that is, to the beauty of a certain harmonious blending of sounds.

THE COMPLETE PROCESS OF HARMONIC ANALYSIS

A curve having been provided, such as the photograph of a sound wave, an electric oscillogram, a diagram of barometric pressures, or a chart of temperatures, its complete analysis by the Fourier harmonic method may be conveniently carried out in accordance with the following scheme:

(*a*) The curve is redrawn to the standard scale required by the Henrici analyzer, so that the wave length is 400 millimeters. Time required: five minutes.

(*b*) The curve is traced with the analyzer, one tracing giving five sine and five cosine coefficients of the complete Fourier equation of the curve, determining five components.

By changing the wire attached to the tracer from one set of pulleys on the integrators to another set, a second tracing gives five more pairs of coefficients, determining ten components of the curve. Continued tracings will give fifteen, twenty, twenty-five, and thirty components. Time required: for each tracing, including making the record, five minutes; for ten components, ten minutes.

(*c*) Each pair of sine and cosine terms as given by the analyzer is reduced by the triangle machine, to determine the true amplitude of the corresponding component, together with its phase. Time required: for ten pairs of terms, including making the record, five minutes.

(*d*) The correctness and completeness of the analysis are verified by setting the synthesizer for the values of the amplitudes and phases of the several components and then reproducing the original curve. Time required: for ten components, five minutes; for thirty components, twelve minutes.

(*e*) The numerical quantities of the analysis are preserved on cards suitable for filing; the synthetic curve is drawn, superposed on the original curve, forming a permanent record of the degree of approximation secured. Time required: included in the time given for the operations (*b*), (*c*), (*d*).

(*f*) The synthesizer may be used to draw each component separately, in its true amplitude and phase. Time required: for ten components, twenty minutes.

(*g*) The true axis of the curve may be determined with the planimeter. Time required: three minutes.

The times mentioned for the several operations are those required when a number of curves are being analyzed in routine; if a single curve is analyzed by itself, a longer time will be consumed. The analysis of a curve as ordinarily understood involves only the operations (*b*) and (*c*), requiring about fifteen minutes for ten components.

Card forms, Fig. 97, 5 by 8 inches in size, have been arranged for preserving the data of analysis and reduction, as required in the study of sound waves. One card contains the data for ten components. The cards for the first ten components, $n = 1$ to $n = 10$, are white in color; for a larger number of components, cards of different colors are used, buff for values of n from 11 to 20, and salmon for $n = 21$ to $n = 30$; blue cards are used for additional information, averages, etc. The data relating to any component are given in the vertical column under the value of n corresponding to the order of the component.

Example of Harmonic Analysis

As a further illustration of harmonic analysis, let it be required to analyze the curve of an organ-pipe tone, shown

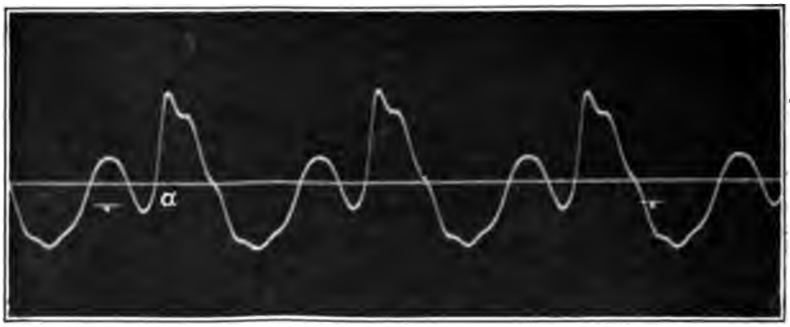


Fig. 96. Photograph of the sound wave from an organ pipe.

in Fig. 96. The curve is traced twice with the analyzer, the necessary change in the wire being made between the two tracings; then each of the ten pairs of sine and cosine terms is reduced with the triangle machine for finding the resultant amplitudes and phases. The actual time required for the complete determination of the ten amplitudes and ten phases, including the recording of the results on the analysis card, was thirteen minutes. (The actual analysis of this curve was extended to twenty components, but it was found to contain only twelve components of appreciable size.)

Fig. 97 is a reproduction of the card containing the com-

CASE SCHOOL OF APPLIED SCIENCE — DEPARTMENT OF PHYSICS

ANALYSIS OF SOUND-WAVES

No. 1690 — Source Organ Pipe—"Reedless Oboe." — Date April 23, 1915

1-10 — Tone C_3 — Abs. N 260 — Purpose Analysis

component, n	1	2	3	4	5	6	7	8	9	10
na_n	+ 22.6	+ 99.6	+ 101.1	+ 76.1	+ 44.5	− 49.1	+ 44.8	+ 24.9	− 11.5	− 7.1
nb_n	+ 94.1	− 87.2	− 42.5	− 7.5	− 26.1	− 10.6	− 3.8	− 66.7	− 36.8	− 21.7
nA_n	96.5	132.0	109.4	76.8	51.6	50.3	45.0	71.1	38.7	22.8
k_n, ( )										
nA_nk_n										
$[nA_nk_n]'$										
A_n	86.5	66.0	36.5	19.2	10.3	8.4	6.4	8.9	4.3	2.3
P_n	76.°	319.°	337.°	354.°	330.°	347.°	354.°	290.°	252.°	252.°
A_nk_n										
Amplitude, %										
Phase	0.°	167.°	109.°	50.°	310.°	250.°	182.°	41.°	288.°	211.°
Intensity, %										

Remarks — Sum — Analyzed [illegible]

Synthesized

FIG. 97. Card form for the record of the analysis of a sound wave.

plete records of the analysis of the above curve (for the first ten components). The first two lines, na_n and nb_n, are the coefficients (each multiplied by n, the order of the component) of the sine and cosine terms of the Fourier equation, form II, as read from the dials of the analyzer. Each pair of numbers is reduced with the amplitude-and-phase calculator, giving nA_n and P_n; each of the multiple amplitudes, nA_n, is divided by the corresponding value of

n, giving A_n; A_n and P_n are the coefficients and phases of form III of the Fourier equation.

The mathematical equation of the organ-pipe curve (twelve components) is, then, as follows, the wave length being equal to 400:

$$\begin{aligned} y = A_0 &+ 96.5 \sin(\theta + 76°) + 66.0 \sin(2\theta + 319°) \\ &+ 36.5 \sin(3\theta + 337°) + 19.2 \sin(4\theta + 354°) \\ &+ 10.3 \sin(5\theta + 330°) + 8.4 \sin(6\theta + 347°) \\ &+ 6.4 \sin(7\theta + 354°) + 8.9 \sin(8\theta + 290°) \\ &+ 4.3 \sin(9\theta + 252°) + 2.3 \sin(10\theta + 252°) \\ &+ 2.2 \sin(11\theta + 230°) + 1.5 \sin(12\theta + 211°) \end{aligned}$$

The graphic interpretation of this equation is given in Fig. 98. The equation as a whole is represented by the original curve at the top; each of the twelve sine terms corresponds to one of the simple curves 1, 2, . . . 12. The numerical values of the several coefficients (96.5, 66.0, etc.) are the actual amplitudes, A_1, A_2, etc., of the component curves, expressed in millimeters, for a wave length, *ab*, of 400 millimeters. If the curve is drawn to any other scale, the coefficients must be changed in the same proportion as is the wave length. The phases of the several components (76°, 319°, etc.) express the positions of the curves, lengthwise, with respect to the initial line, *ai*.

The line photographed as the axis often is not the mathematical axis of the curve. The true axis is found, as described on page 107, with the planimeter. The curve was traced with the planimeter, showing that the area of that part of the curve which lies below the horizontal line exceeds that above by 2704 square millimeters; this quantity, divided by the wave length, 400, gives 6.75 millimeters as the distance of the true axis below the assumed axis. The true axis is the dotted line *a′b′* in Fig. 98; if the curve is

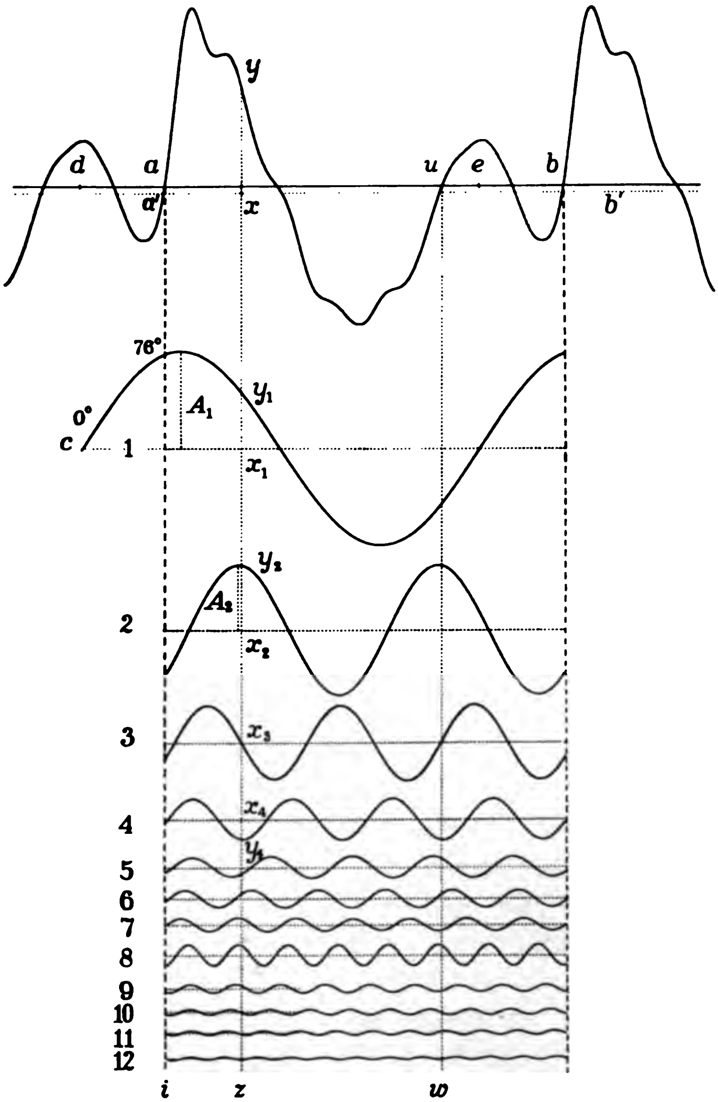


FIG. 98. An organ-pipe curve and its harmonic components.

traced with respect to this line, the areas above and below the line will be found equal. The distance of the true axis from the assumed line, -6.75, is the numerical value of the coefficient A_0 of the equation of the curve.

The analysis means that the original curve is equivalent to the simultaneous sum, or composite, of the several component curves. If the axes of the twelve components all coincided with the axis $a'b'$, then the algebraic sum of the twelve ordinates of the curves at any point x (along the line yz) would be equal to the ordinate xy of the original curve. The ordinate of the first component for the point x is $+x_1y_1$; for the second component it is $+x_2y_2$; for the third it is zero, x_3; for the fourth it is $-x_4y_4$, etc.; the sum is positive and equal to xy. For the point u, the sum of the ordinates of the components on the line uw is zero, that is, the curve crosses the axis at this point. This graphic representation of the analysis of a curve is in accordance with the principles illustrated in the models of three waves shown in Lecture II, page 59. If the separate simple sounds from twelve tuning forks (or other source) produce motions in the air represented by the twelve component curves, then the composite tone of all would produce a composite motion represented by the original curve.

The meaning of the phases (or epochs) of the several components may be further explained by reference to the figure. The starting point for tracing with the analyzer is arbitrarily selected; it may be, for instance, the point a, Figs. 96 and 98, where the photographed curve crosses the photographed axis, which may or may not be the true axis. The phases obtained by analysis then give the relations of the several component curves to the assumed initial line ai. In the example here shown, the phase of the first component is 76° ; this means that where the curve crosses the initial line, the first component has already progressed 76° (one wave length equals 360°) from its own zero point. The zero point for the first component is then $\frac{76}{360}$ of the wave length to the left of the initial line, at *c* in Fig. 98. The phases of the other components have similar interpre-

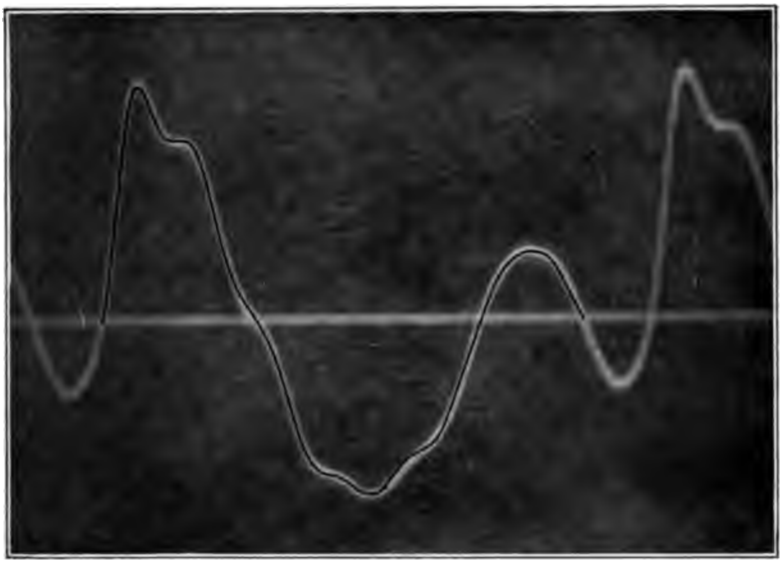


FIG. 99. Proof of the analysis of a curve by synthesis.

tations, each being measured in terms of its own wave length.

It is sometimes desirable to consider the beginning of a curve as the point where the phase of the first component is zero; the initial line would then be at the point *d*, and the wave length would be *de*. This point, in general, is not where the curve crosses the axis and there is no way of determining it in advance of analysis.

The phases of the several higher components at the point where that of the first component is zero are convenient for the comparison of phases; these are obtained by subtracting from each phase, as obtained by analysis, n times the phase of the first component. These relative phases are determined without calculation with the synthesizer as explained on page 114, and they are recorded on the card in the line labelled "Phase" as shown in Fig. 97.

The verification of an analysis is made by synthesis. The equation of the curve is set up on the synthesizer, and the curve is drawn by machine, superposed on the enlarged drawing of the curve which was used with the analyzer. For illustration in this instance, the original photograph has been enlarged and the synthetic curve has actually been drawn by machine on the photograph. Fig. 99 is a reproduction of the original and synthetic curves. The likeness is sufficiently close for general purposes; a more exact reproduction would probably require the inclusion of a very large number of higher components, all of which have very small amplitudes.

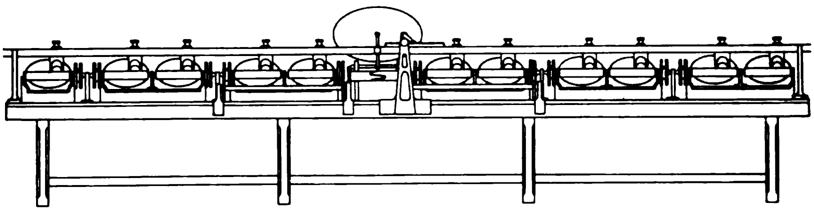


Fig. 100. Kelvin's tidal harmonic analyzer.

Various Types of Harmonic Analyzers and Synthesizers

The general methods of harmonic analysis and synthesis which have been described in detail in the preceding pages are applicable to all kinds of investigations requiring such treatment. However, variations of the mechanical devices are sometimes desired to obtain special results. Brief mention will be made of several other types of instruments.

Probably the first useful application of harmonic analysis was to tidal analyzing and predicting machines by Lord Kelvin, in 1876. The rise and fall of the tides, having been

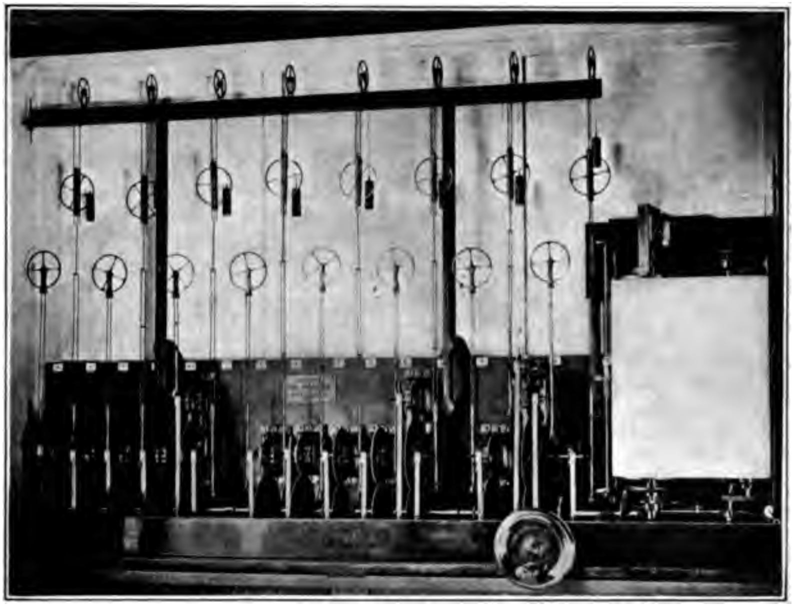


FIG. 101. Kelvin's tide predictor.

observed at a given port for a year or more, is represented by a curve which is then analyzed. The tidal components being known, it is possible to synthesize these for future dates, that is, to predict the tides.[50] Kelvin's analyzer is shown in Fig. 100, and the predictor in Fig. 101.

A tide-predicting machine of remarkable completeness and perfection has recently been constructed by the United

Coast and Geodetic Survey at Washington; Fig. a general view of this instrument.[51]

essor A. A. lson has de- a very ingen- armonic syn- er and ana- for eighty nents, which applied most ely to the of light [52] A Michel- nthesizer, of construction, enty compo- is shown in 03; the ma- also serves as alyzer. The wave form is the edge of t of card or which is then . to set the e; a curve is by means of the ampli- f the required nents may be

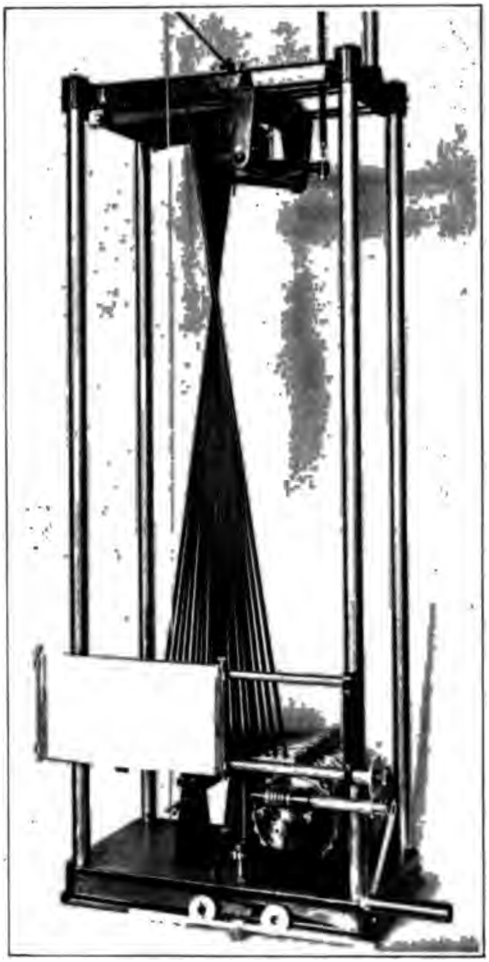


FIG. 103. Michelson's harmonic analyzer and synthesizer for twenty components.

ined in a manner described in the references. nonic analyzers are employed in electrical engineer-

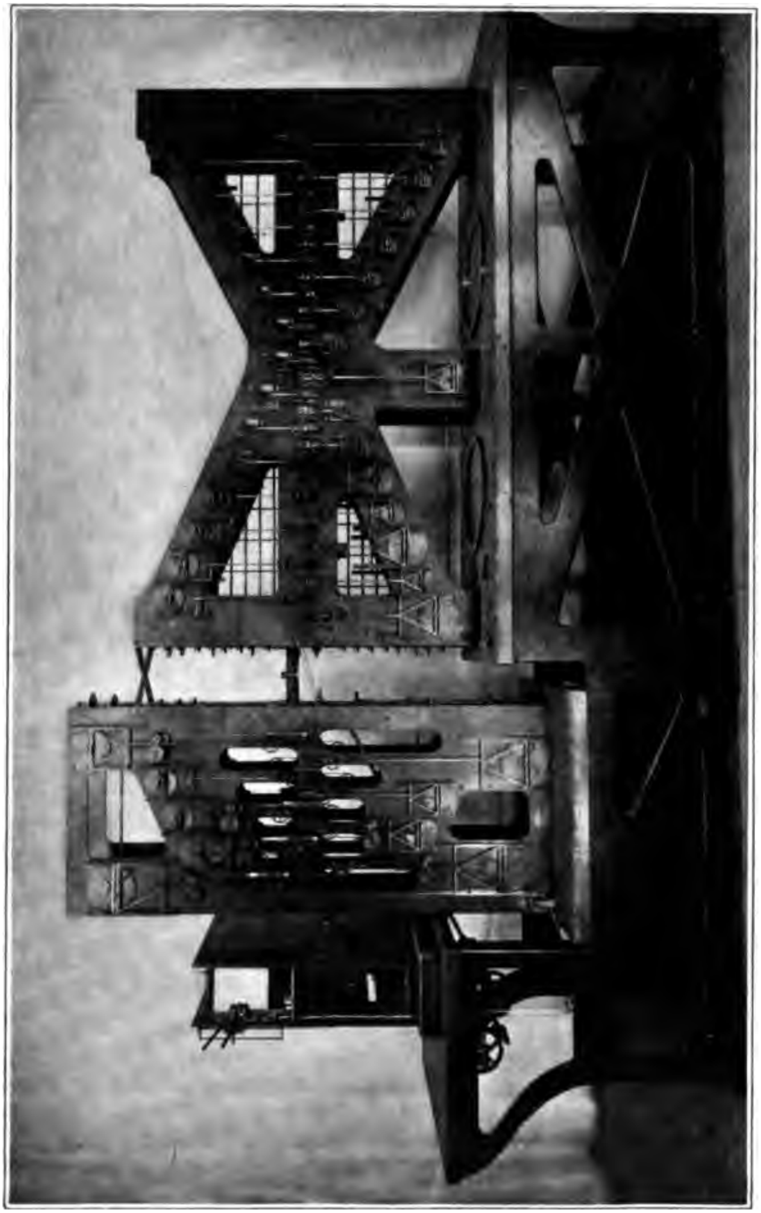


FIG. 102. Tide predictor of the United States Coast and Geodetic Survey.

States Coast and Geodetic Survey at Washington; Fig. 102 is a general view of this instrument.[51]

Professor A. A. Michelson has devised a very ingenious harmonic synthesizer and analyzer, for eighty components, which he has applied most effectively to the study of light waves.[52] A Michelson synthesizer, of recent construction, for twenty components, is shown in Fig. 103; the machine also serves as an analyzer. The given wave form is cut on the edge of a sheet of card or metal, which is then applied to set the machine; a curve is drawn by means of which the amplitudes of the required components may be determined in a manner described in the references.

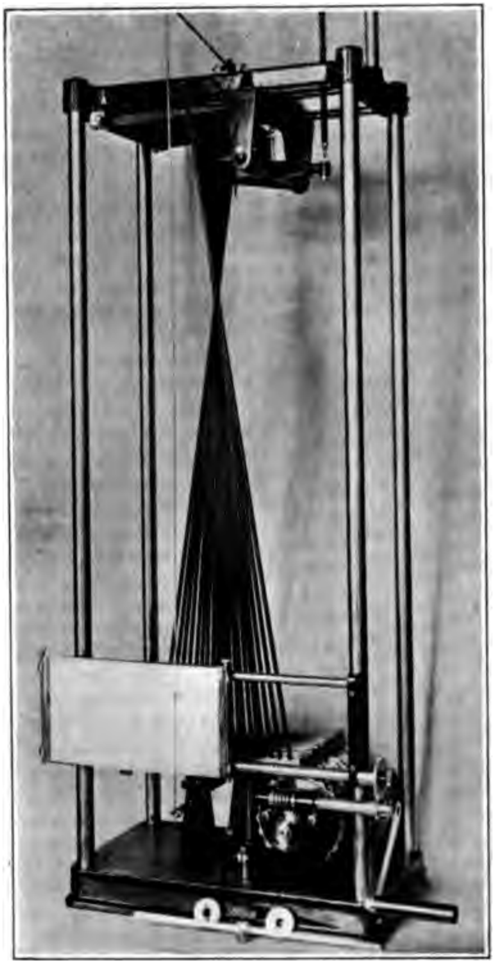


FIG. 103. Michelson's harmonic analyzer and synthesizer for twenty components.

Harmonic analyzers are employed in electrical engineering for the study of alternating-current waves and other periodic curves. The curves being investigated often have few components, or the interest is centered in a few components; in such cases simpler forms of analyzers may be used in which the integrating is performed by a planimeter of the ordinary type. Figures 104, 105, and 106 show instruments of this kind designed by Rowe, Mader, and Chubb, respectively.[53] These machines may be used with a wave of any size, such as the original oscillogram; the curve is traced with the stylus, giving one component; by changing one or more gear wheels and again tracing, another component is found, and so on.

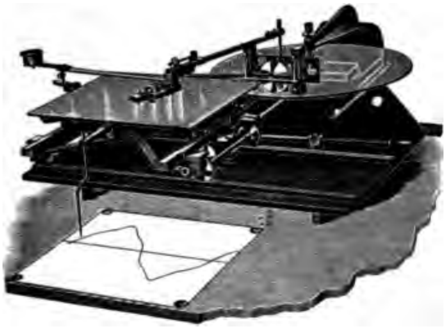


FIG. 104. Rowe's harmonic analyzer.

FIG. 105. Mader's harmonic analyzer.

Several other types of harmonic analyzers are described in Horsburgh's "Instruments of Calculation" and in Morin's "Les Appariels d'Intégration." [54] These books also describe many subsidiary instruments and processes which are helpful in numerical work.

Fig. 106. Chubb's harmonic analyzer.

Arithmetical and Graphical Methods of Harmonic Analysis

Mention has already been made of the application of harmonic analysis to the study of acoustics, the tides, electricity, optics, and mathematics. The method is also useful in the investigation of other more or less periodic phenomena. In meteorology it is applied to the study of hourly or daily temperature changes, barometric changes, etc.[55] In astronomy the periodicity of sun spots, magnetic storms, variable stars, etc., may be treated by harmonic analysis.[56] In mechanical engineering, valve motions and other mechanical movements may be investigated.[57] The method is also used in geophysics, in naval architecture, and in the study of statistics.[58]

When the number of curves to be analyzed is small and especially when the number of components is limited, it may not seem necessary to provide a machine for perform-ing the analyses. While the general solution of the problem was given by Fourier in his original work "La Théorie Analytique de la Chaleur" (Paris, 1822),[45] yet the labor involved in the numerical reduction is very great. Many arithmetical and graphical schemes for facilitating the work have been developed by Wedmore, Clifford, Perry, Kintner, Steinmetz, Rosa, Runge, Grover, S. P. Thompson, and others. In general a set of coördinates of the curve is measured, and these measures are reduced in accordance with the scheme selected to give the amplitudes and phases of the components.

Steinmetz gives general formulæ systematically arranged for the calculation of any number of components, of odd or even order.[59] Several numerical examples are given, selected from electrical engineering, while another is the determination of the first seven components of a diagram of mean daily temperatures.

Runge's method depends upon a scheme of grouping the terms so as to facilitate the numerical work.[60] A number of ordinates of the curve, n, are measured. For odd components only, the ordinates are evenly distributed over a half wave and give $\frac{1}{2}n$ components; for odd and even components the ordinates are evenly distributed over the whole wave, and give $\frac{1}{2}n - 1$ components. Runge gives schemes for 12, 18, and 36 ordinates. Bedell and Pierce give a scheme and an example for determining the odd components from 18 ordinates.[61] Carse and Urquhart give the scheme with numerical examples for odd and even components from 24 ordinates.[54] F. W. Grover gives six schedules according to Runge's method with examples for the calculation of the odd components from 6, 12, and 18 ordinates; and also schedules for both odd and even components from 6, 12, and 18 ordinates.[62] There is also given a special multiplication table for all the sine and cosine products required. When one of these schedules is applicable, the method as described by Grover is probably the most expeditious available for numerical analysis. H. O. Taylor has developed a convenient method for constructing complete schedules adapted to general or special conditions.[62]

S. P. Thompson has provided several schedules also based on Runge's method, which are very expeditious. More recently he has developed an approximate method of harmonic analysis in which all multiplication by sines and cosines is dispensed with, and only a few additions and subtractions of the numerical values of the ordinates is required.[63] The method is applicable only to periodic curves in which the components higher than those being calculated are absent; if higher components are present, their values may be added to those of the lower components in certain cases. Thompson gives schedules for the first three components, suitable for the analysis of valve motions, a schedule for the first seven components, and one for the odd components to the ninth, and a special schedule suitable for tidal analysis.

A large number of graphical methods for harmonic analysis have been devised. These are suitable for curves having only a few components, but it is doubtful whether they are any more expeditious than the equivalent arithmetical methods, and usually they are not so precise. A convenient graphical method is that devised by Perry.[64] Numerous other methods are described in "Modern Instruments of Calculation" and in the volumes of the *Electrician*.[65]

For comparison the curve which was analyzed by machine in 13 minutes, as described on page 122, was analyzed by Steinmetz's method, requiring about 10 hours to obtain ten components, and by Grover's method the time was about 3 hours for eight components odd and even (the largest number for which a scheme is arranged). (Grover mentions that by his method eight odd components can be determined in less than an hour by one familiar with the process.) The curve shown in Fig. 76, page 100, which was known to have but three components, was analyzed by Thompson's short method, the three amplitudes and phases being evaluated in fifteen minutes. The analysis of the same curve (for five components) by machine required less than seven minutes.

Analysis by Inspection

A familiarity with the effects produced by various harmonics on the shape of a wave will often enable one to judge by inspection what harmonics are present.

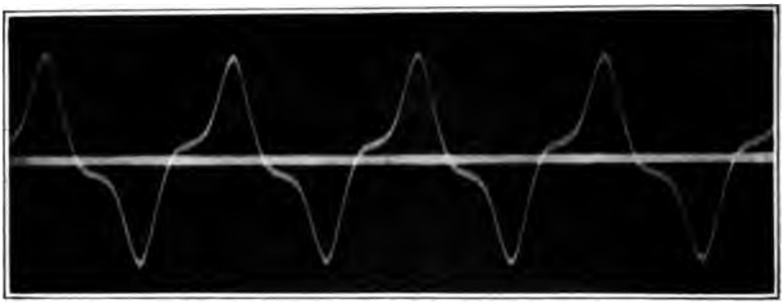


Fig. 107. Symmetrical wave form of an electric alternating current.

If a wave consists of alternate half-waves which are exactly of the same shape but opposite in direction, that is, if the wave is a symmetrical one with respect to its axis, it can contain only odd-numbered components; if a wave is not symmetrical, it must contain some even-numbered components, and it may contain both odd and even. Sound waves belong to the latter class, no instance having been observed of a symmetrical sound wave, except that of a tuning fork which has only one component and is a simple sine curve. Electric alternating-current waves are usually of the first kind, containing only odd-numbered components; such a wave is shown in Fig. 107.

In some instances a particular high partial may be prominent and so impress its effect on the wave as to produce

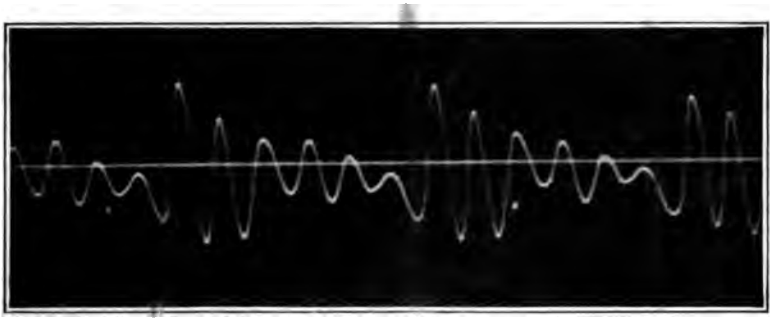


FIG. 108. Photograph of the vowel *a* in *father*, intoned upon the pitch $n = 159$.

distinct wavelets or ripples on the main wave form even though this is itself irregular; the order of such a partial is at once determined by merely counting the number of such wavelets occurring in one fundamental wave length. Fig. 108 shows a wave for the vowel *a* in *father;* this curve is evidently complex, but there are six distinct sub-peaks on one wave, and the sixth partial is prominent. Since the frequency of the fundamental is known to be 159, that of the sixth partial is 954. Analysis shows that the first ten components of this curve have the following amplitudes, corresponding to a wave length of 400:

I = 4	II = 15	III = 18	IV = 12	V = 20
VI = 60	VII = 21	VIII = 16	IX = 2	X = 3

An instance where analysis by inspection is sufficient is given in Fig. 136, on page 187, which shows the wave from a tuning fork having but one overtone. The partial is prominent and produces very definite sub-peaks; it is found that there are about twenty-five wavelets to four large waves, that is, the partial has a frequency about 6.25 times that of the fundamental and is inharmonic.

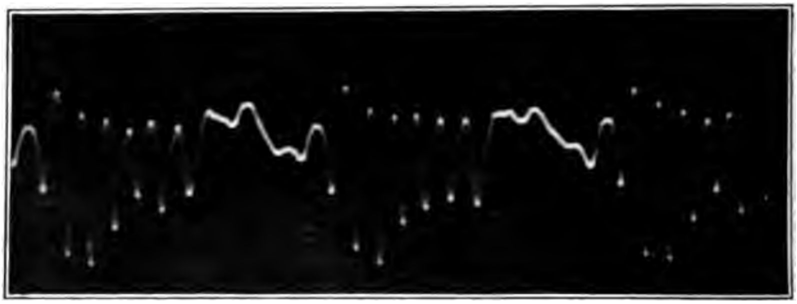


FIG. 109. Clarinet wave showing beats produced by the higher partials.

In some instances the peaks due to the partials are very pronounced in portions of a wave and almost disappear in other parts; this indicates that there are beats between certain partials. If there is one beat per wave length, it is produced by two adjacent partials; if there are two beats, then the orders of the partials differ by two. The average distance between sub-peaks is found by measurement and, when compared with the wave length, gives the average order of the prominent partials, from which it is then usually possible to specify their exact orders. While this method is most useful for waves having a few components, such as alternating-current waves, it may be applied to a wave as complex as that of the tone of a clarinet, Fig. 109. Since there is one beat per wave, there are two prominent adjacent overtones. Actual measurement of these wavelets shows an average length of $3\frac{1}{3}$ millimeters; the fundamental measures 38 millimeters, about $11\frac{1}{2}$ times the length of the sub-wave; the conclusion is that the eleventh and twelfth partials are producing the beat. The correctness of this conclusion is proved by the actual analysis which gives the following values for the first twelve components of the curve, corresponding to a wave length of 400:

I = 29	II = 7	III = 20	IV = 1	V = 2	VI = 6
VII = 6	VIII = 8	IX = 16	X = 9	XI = 30	XII = 35

A photograph of the sound of the explosion of a sky-rocket in a Fourth of July celebration is shown in Fig. 110.

FIG. 110. Photograph of the sound of the explosion of a skyrocket.

The rocket was about a quarter of a mile from the recording apparatus, the sound entering the laboratory through two open windows. While this curve as a whole is not periodic, yet two or more periodicities are clearly shown. The time signals are $\frac{1}{100}$ second apart, and comparison shows one frequency of about 130 per second producing the principal feature of the curve; superposed upon this is a much higher frequency of about 2000 per second.

Periodic and Non-periodic Curves

The Fourier analysis is suitable and complete for any curve whatever within the distance called one wave length, even though there is no repetition of this form; if the portion analyzed is successively and exactly repeated, that is, if the curve is periodic, it represents a wave motion and the analysis represents the entire wave. A periodic wave is shown in Fig. 111, which is a photograph of the vowel *a* in *mat*.

If a curve representing some physical phenomenon is periodic, then each separate term of the Fourier equation

Fig. 111. A periodic curve; a photograph of the vowel *a* in *mat*.

of the curve may be presumed to correspond to something which has a physical existence; it is the belief in this statement, amply supported by investigation, which leads one to analyze sound waves by this method; as explained under Ohm's Law, page 62, each term is presumed to correspond to a simple partial tone which actually exists.

If the curve representing the physical phenomenon is non-periodic, any portion of the curve may be analyzed, and it will be completely represented as to *form* by the Fourier equation, within the limits analyzed, *but not beyond these limits*. In this case, the separate terms of the Fourier series may not correspond to anything having a separate physical existence; in fact the equation may be presumed to be merely an artificial mathematical formula for the short irregular line which has been analyzed. The analysis of the profile portrait, described on page 119, illustrates this application of Fourier analysis; there is no periodicity of the wave form, and the separate terms of the equation can have no real significance.

There is no general method for analyzing non-periodic curves, that is, for curves containing incommensurable (inharmonic) or variable components; such a method is very much desired for the study of noises and of sounds from such sources as bells, whispered words, the consonant

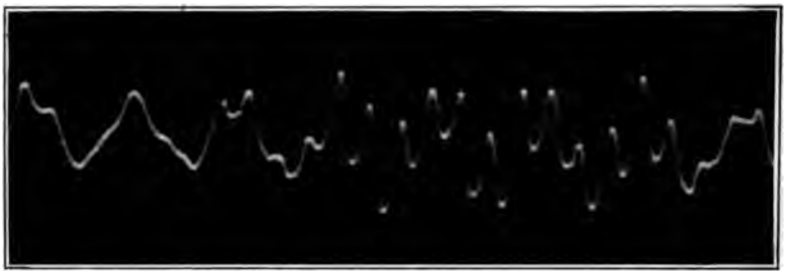


Fig. 112. A non-periodic curve; a photograph of the sound from a bell.

sounds of speech, and in fact all speech sounds except the simple vowels; this need is probably the greatest of those unprovided for.[56, 58] A non-periodic curve, a photograph of the sound from a bell, is shown in Fig. 112; there is no apparent wave length in this curve, and an analysis of any portion of it would probably give an equation containing an infinite number of terms, though the real sound is undoubtedly compounded from a finite number of partials which are inharmonic and therefore indeterminate. Much information may be obtained from such curves by making skillfully assumed analyses.

LECTURE V

INFLUENCE OF HORN AND DIAPHRAGM ON SOUND WAVES, CORRECTING AND INTERPRETING SOUND ANALYSES

Errors in Sound Records

The photographs and analyses of sound waves obtained by the complicated mechanical and numerical processes described in Lectures III and IV, are, unfortunately, not yet in suitable form for determining the tone characteristics of the sounds which they represent. Before these analyses can furnish accurate information they must be corrected for the effects of the horn and the diaphragm of the recording instrument, a correction involving fully as much labor as was expended on the original work of photography and analysis. For the sake of greater emphasis, it may be directly stated that the neglect of the corrections for horn and diaphragm often leads to wholly false conclusions regarding the characteristics of sounds, since horns and diaphragms of different types give widely differing curves for precisely the same sound.

For research upon complex sound waves, a recording instrument using a diaphragm should possess the following characteristics: (*a*) the diaphragm as actually mounted should respond to all the frequencies of tone being investigated; (*b*) it should respond to any combination of simple frequencies; (*c*) it should not introduce any fictitious frequencies; (*d*) the recording attachment should faithfully transmit the movements of the diaphragm; and (*e*) there must be a determinate, though not necessarily simple, relation between the response to a sound of any pitch and the loudness of that sound.

It is well known that the response of a diaphragm to waves of various frequencies is not proportional to the amplitude of the wave; the diaphragm has its own natural periods of vibration, and its response to impressed waves of frequencies near its own is exaggerated in degrees depending upon the damping. The resonating horn also greatly modifies waves passing through it. Therefore, it follows that the resultant motion of the diaphragm is quite different from that of the original sound wave in the open air.

The theory of these disturbances for simple cases is complete, but what actually happens in a given practical apparatus is made indeterminate by conditions which are complicated and frequently unknown. There being no available solution of this problem, it was necessary to make an experimental study of these effects as they occur in the phonodeik.[66]

It has been proved that the phonodeik possesses several of the characteristics mentioned; (*a*) it has been shown by actual trial that it responds to all frequencies to 12,400; (*b*) various combinations of simple tones up to ten in number have been actually produced with tuning forks, and the photographic records have been analyzed; (*c*) in each case the analysis shows the presence of all tones used, and no others. We have then only to determine the accuracy with which the response represents the original tones, the qualifications (*d*) and (*e*) mentioned above; this requires the investigation of all the factors of resonance, interference, and damping.

Ideal Response to Sound

In Lecture II it has been explained that the intensity or loudness of a simple sound is proportional to the square of the amplitude multiplied by the square of the frequency, that is, to $(nA)^2$. A recording apparatus having ideal response must fulfill the following condition: let all the

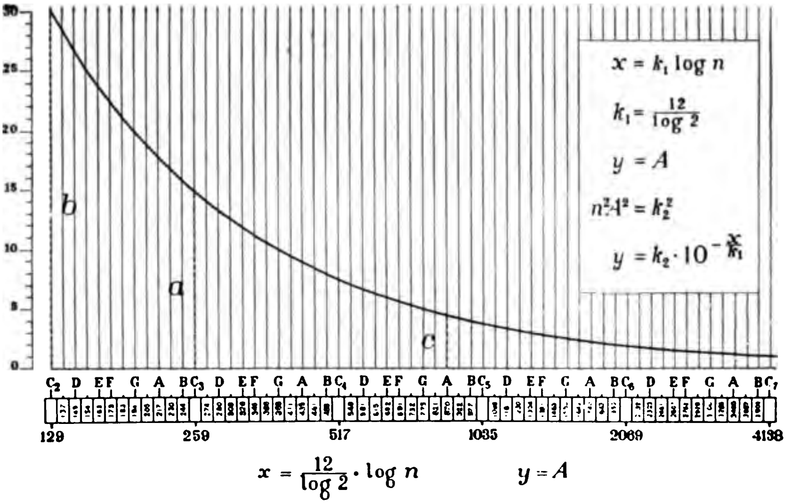


FIG. 113. Curve showing the amplitudes for a sound of varying pitch but of constant loudness.

tones of the musical scale (simple tones) from the lowest to the highest, and *all exactly of the same loudness*, be sounded one after the other and be separately recorded; let the amplitudes of the various responses be measured, and each amplitude be multiplied by the frequency of the tone producing it; then the *squares* of the products of amplitude and frequency must be constant throughout the entire series.

A curve of ideal amplitudes is given in Fig. 113, the vertical scale of which is one of linear measure, centimeters for instance, and the horizontal scale is a logarithmic scale of frequencies. The divisions represented by the light vertical lines correspond to the successive tones of the musical scale, as is explained later in this lecture. The properties of this curve are as follows: if a simple sound having the pitch $C_3 = 259$ produces a record which has an amplitude represented by the ordinate *a* of the curve, then a sound of exactly the *same loudness*, but one octave lower in pitch, is correctly represented by a record the amplitude of which is the ordinate *b*; further, the sound A_4, having 870 vibrations per second, and of the same loudness as either of the others, should produce an amplitude measured by the much shorter ordinate, *c*; and similarly for any note of the scale.

Actual Response to Sound

The determination of the actual response of a recording apparatus requires a set of standards of tone intensity for the entire scale of frequencies under investigation. The practical fulfillment of this requirement for a time seemed an impossibility. A manufacturer of organ pipes who became interested in the problem provided two complete sets of pipes, an *open diapason* of metal and a *stopped diapason* of wood, especially voiced and regulated to uniform loudness throughout, according to his skilled judgment. The stopped diapason pipes, Fig. 114, sixty-one in number, range in pitch from $C_2 = 129$ to $C_7 = 4138$; the scale is extended by nineteen specially voiced metal pipes to a pitch 12,400. The adjustment of these pipes for uniform loudness has been improved and verified by two experimental methods. While this scale is arbitrary and of moderate precision, it is the only available method by which progress has been possible, and its use has led to most interesting results in studying the responses and correction factors of the phonodeik under various conditions.

The sounds from the several pipes of the two sets have been photographed and analyzed. The analyses show that the open diapason pipes have a strong octave accompanying the fundamental, while the stopped pipes give practically simple tones; the latter are used exclusively in obtaining the correction factors as explained later.

These pipes are sounded in front of the phonodeik, one

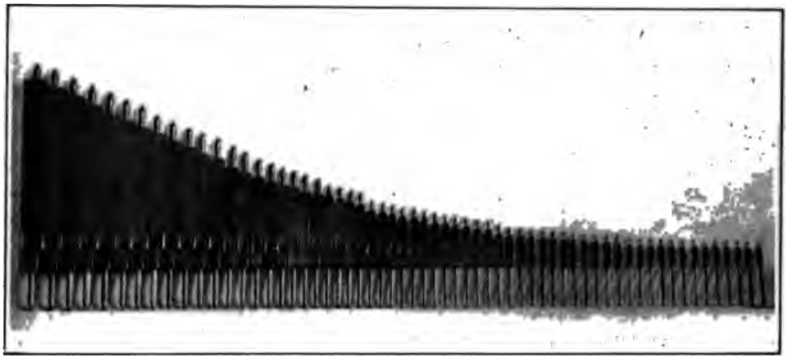


FIG. 114. A set of organ pipes of uniform loudness.

at a time, and the resulting amplitudes of vibration of the diaphragm are recorded photographically. The film is stationary; the first pipe, C_2, is sounded steadily; the shutter is released, giving an exposure of about $\frac{1}{40}$ second; the spot of light which is vibrating back and forth in a straight line falls on the film, making several excursions within the time of exposure, and records the amplitude of the vibration; the record for the first pipe is *C* in Fig. 115. The film is moved lengthwise about a quarter of an inch, and while the second pipe, $C_2\sharp$, is sounding, the resulting amplitude is photographed. The process is continued until the amplitudes produced by the sixty-one pipes are recorded. The vertical scale of such a chart represents linear amplitude, while the horizontal scale is a logarithmic scale of frequencies which is described on page 168.

A curve may be drawn through the upper ends of these amplitude records, showing the "responsivity" of the apparatus under the conditions of the experiment. Fig. 115 shows the responses used in correcting the analysis of the organ-pipe curve shown on page 122, while the interpretation of the responses is given on page 163.

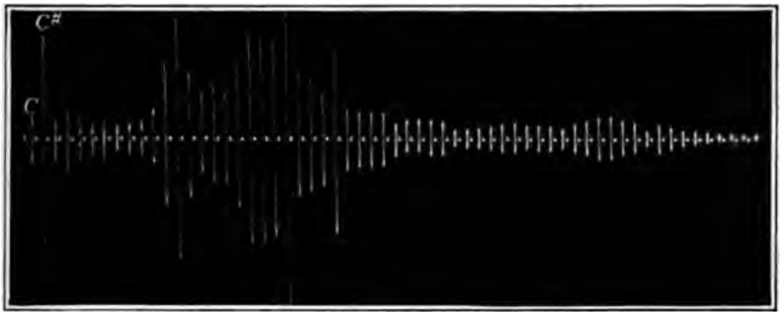
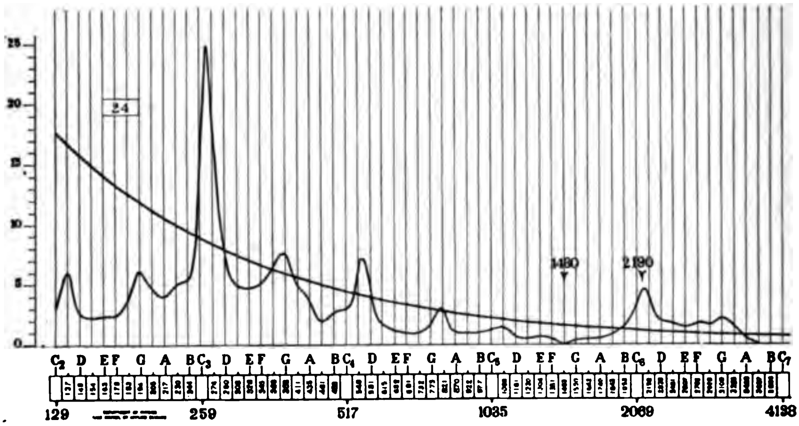


FIG. 115. Photographic record of the amplitudes of vibration for the organ pipes of uniform loudness.

The irregular curve of Fig. 116 is the response obtained with one of the earliest forms of phonodeik; it shows an almost startling departure from the ideal response represented by the smooth curve. What produces the range of mountains, with sharp peaks and valleys? Why is there no response for the frequency 1460; why is it excessive for frequencies from 2000 to 3000? There were five suspected causes: (1) unequal loudness of the pipes; (2) the diaphragm effects; (3) the mounting and housing of the diaphragm; (4) the vibrator attached to the diaphragm; (5) the horn.

The investigation of these peculiarities was most tantalizing; the peaks acted like imps, jumping about from place to place with every attempt to catch them, and chasing and pushing one another in a very exasperating manner. Perhaps two months' continuous search was required to find the causes of "1460" and "2190" alone. The investigations led to many improvements in the phonodeik and to



$$x = \frac{12}{\log 2} \cdot \log n \qquad y = A$$

FIG. 116. A response curve obtained with an early form of phonodeik.

a practical method of correction for the departures from ideal response.

RESPONSE OF THE DIAPHRAGM

Experiments have been made with diaphragms of several sizes and thicknesses, and of various materials, such as iron, copper, glass, mica, paper, and albumen. The experiments described in this section concern circular glass diaphragms having a thickness of 0.08 millimeter, and held around the circumference, either firmly clamped between hard cardboard gaskets and steel rings, or loosely clamped between soft rubber gaskets; the diaphragm is entirely free, there being no horn or housing of any kind. The silk fiber of the phonodeik vibrator is attached to the diaphragm

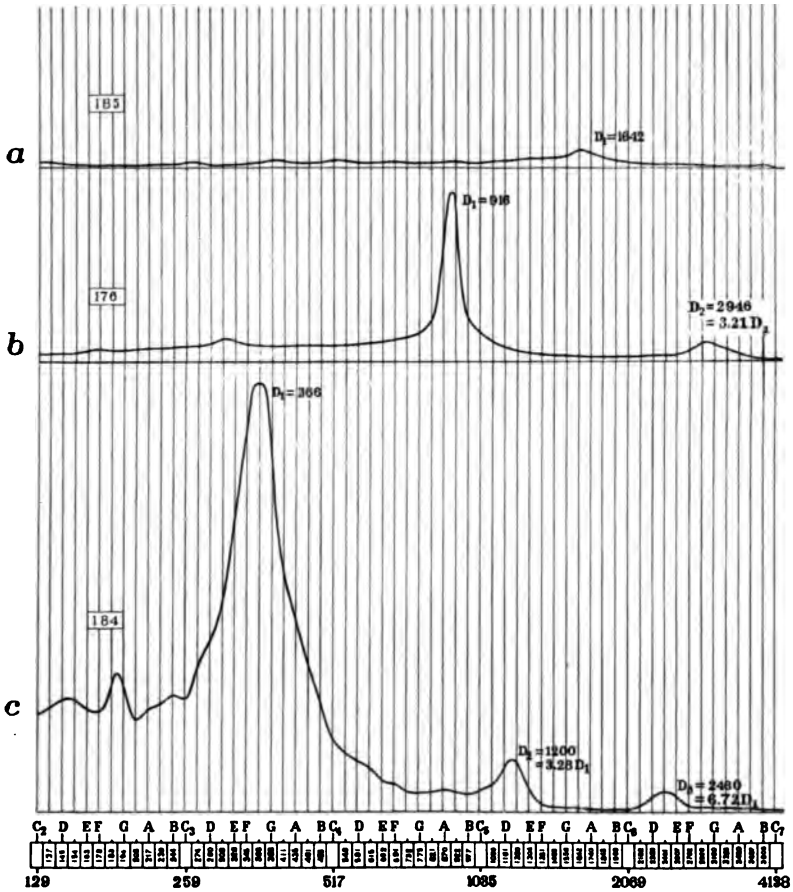


FIG. 117. Resonance peaks for diaphragms of different diameters.

to record its movements. The pipes already described were sounded in succession in front of the diaphragm, and observations were made of the response under various conditions. The responses of three glass diaphragms of 22, 31, and 50 millimeters diameter, respectively, are shown in *a*, *b*, and *c*, Fig. 117.

When the pitch of the pipe being sounded is near the natural frequency of the diaphragm, the latter moves easily and responds vigorously; the diaphragm is in sym-

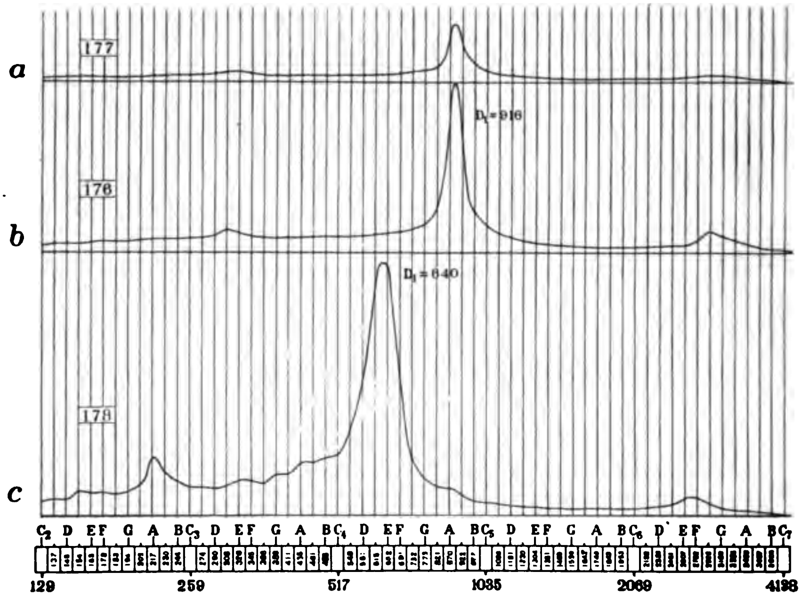


FIG. 118. Effects of clamping and distance on the response of a diaphragm.

pathy or in *resonance* with the tone; the response curve shows a peak for such a resonance condition. The natural period of the largest diaphragm had a frequency of 366, corresponding to which there was a large response, as shown in the lower curve of the figure. Two other peaks represent the natural overtones of the diaphragm; these overtones have frequencies 3.28 and 6.72 times that of the fundamental, and are inharmonic. The other curves show that the natural period of the diaphragm rises in pitch as the diameter decreases, and that the actual response becomes less.

The lower curve, *c*, Fig. 118, is the response of a glass diaphragm 31 millimeters in diameter, held lightly in the clamping rings. When the clamping is tightened, the response is as shown in the middle curve; the natural period is increased from 640 to 916, while the amplitude is reduced from 228 to 160. The upper curve shows the response for the same diaphragm as for the middle curve, but with the pipes at a greater distance; the curve is of the same general shape, while the response is of diminished amplitude.

Chladni's Sand Figures

Chladni's method of sand figures has been employed in studying the conditions of vibration of the diaphragm.[67] A plate or diaphragm, clamped at the edge or at an interior point, may be made to vibrate in many different modes. When sand is strewn on the plate it is observed that portions are moving up and down, throwing the sand into the air. There are certain lines toward which the sand gathers, indicating that these parts are relatively at rest; the lines on which the sand accumulates are called *nodal lines,* and form patterns or figures which are always the same for the same note, but differ for each change of pitch or quality. It is thus shown that a diaphragm vibrates in various subdivisions.

A diaphragm of glass held in circular rings was placed horizontally, the vibrator being attached to the under side; sand was then sprinkled over the diaphragm which was made to respond in succession to each one of the eighty pipes of frequencies from 129 to 12,400. The characteristic nodal lines produced in each instance were either sketched or photographed.

When the test sound has a pitch equal to the natural frequency of the diaphragm, 366, the diaphragm vibrates

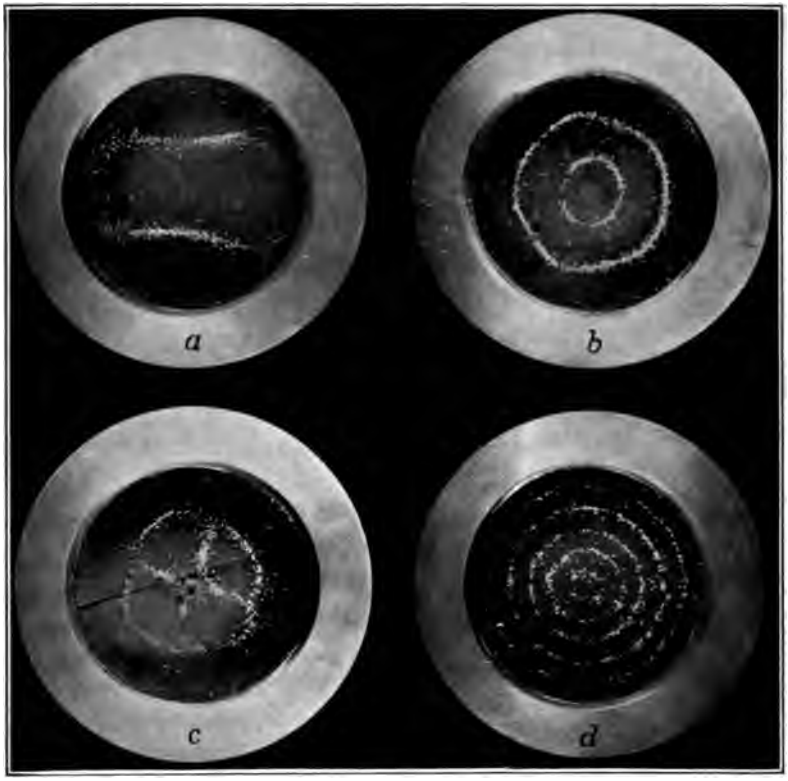


FIG. 119. Different modes of vibration of a diaphragm, shown by sand figures.

as a whole vigorously; there are no nodal lines, except the circumference, and even this is probably in motion when the clamping is light. There are no nodal lines for tones within the octave lower and the octave higher than the natural frequency.

As the pitch of the test sound rises, the area of the plate which can respond seems to be less than the whole, and this part moves, with the formation of a nodal boundary line of more or less irregular shape; Fig. 119, *a*, is the photograph of the pattern for the frequency 977. Since parts of the plate are now at rest, no part can vibrate through a large amplitude, and the response is greatly diminished, as shown in the response curve, Fig. 117. A second maximum response is obtained from a sound corresponding to the first overtone of the diaphragm having a frequency of 1200, about 3.28 times that of the fundamental; this is represented by the second, smaller peak of the curve. The nodal figure on the diaphragm is a circle of medium size.

As the pitch of the test sound rises, the figures again become irregular and of smaller area, and two concentric nodal circles appear, *b*, Fig. 119, corresponding to the second overtone, of a frequency of 2460, 6.72 times that of the fundamental. As the pitch rises still higher, the areas become smaller, with the formation of three, four, and five concentric circles, and other designs. The photographs *c* and *d* show the nodal lines for frequencies of 2600 and 10,400.

Free Periods of the Diaphragm

Besides the two methods already described, one by direct measure of the response, the second by means of the Chladni sand figures, a third method of determining the diaphragm characteristics has been used, that of photographing the free-period effects. The diaphragm is given a single displacement, and upon release is allowed to vibrate freely. This displacement may be produced by the noise of a handclap, or by attaching a fine thread to the diaphragm which is gently pulled aside and the thread then burned. The frequency is given by comparison with the time signals.

The curve *a*, Fig. 120, was obtained with a free, uncovered diaphragm; two distinct frequencies are shown at the beginning of the motion, 1000 and 3100; the latter persists for $\frac{1}{200}$ second, while the former lasts about $\frac{2}{100}$ second. Curve *b* was obtained when the diaphragm was inclosed in a housing forming a front and back cavity, but with no horn; the air cushions damp the vibrations, there being

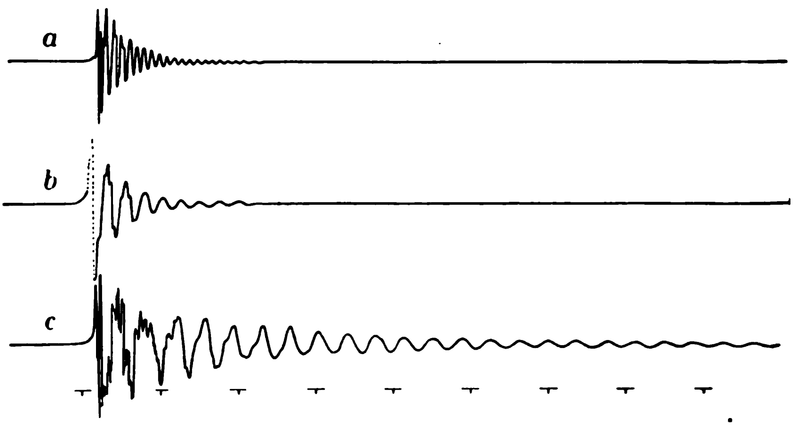


Fig. 120. Free-period vibrations of a diaphragm.

nine vibrations now, while before there were twenty-two; the frequency of the diaphragm has been reduced to 400, and there is a higher frequency of 2190 due probably to the natural period of the air in the chambers. When a horn is added, the curve *c* is obtained; the frequency of vibration of the air in the horn is 264, and it continues to vibrate for about a tenth of a second; the frequency of the diaphragm is now 530, and that of the back cavity is 2190; these vibrations die out in about $\frac{2}{100}$ second, as before.

Influence of the Mounting of the Diaphragm

In the early experiments it was thought desirable, in order to protect the diaphragm from indirect sounds, to inclose it in a housing; various shapes and sizes of front and back coverings, shown in Fig. 121, were tried. The diaphragm is in effect between two cavities, and it was found that each produces its own complete and independent resonance effects, and that these influence each other through the diaphragm. When the frequencies of these cavities are in certain ratios, the response of the diaphragm is annulled by interference effects; at other times these cavities produce exaggerated responses.

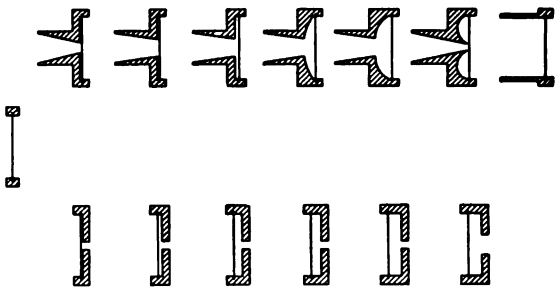


Fig. 121. Shapes of front and back coverings for a diaphragm.

Experiment indicated that the back should be uncovered, since the effect of a sound produced in front of the horn is ordinarily of no influence on the back of the diaphragm. The front must, of course, be covered and the connections between the cover and the horn, and the cover and the diaphragm, must be air-tight. The best results were obtained by using a shallow cup-shaped front cover with an opening for the horn, which may have a diameter about one fourth of that of the diaphragm. If the front cover is close to the diaphragm, the damping effect of the air cushion may be too great.

Influence of the Vibrator

Elaborate studies have been made of the influence of the vibrator on the response of the apparatus; among the factors investigated are the mass and shape of the vibrator, size and shape of the mirror, material and length of the fiber, material and length of the tension piece and its hysteresis and damping effects, amount of tension, moduli of elasticity, and temperature; computations also have been made of the inertias, accelerations, forces, and natural periods, of the various parts, and their resonances and interferences for frequencies up to 10,000; the differential equations of motion of the actual system have been formed and solved.

It is quite out of place to explain the details of this work here; the final practical result is the demonstration that for frequencies less than 5000 the vibrator produces no appreciable effect on the record.

This conclusion is verified by the results of a further study with the Chladni sand figures; besides the set of figures described previously, with the vibrator attached to the diaphragm, a second complete set of figures was obtained without the vibrator; the two sets are practically identical except for a shifting of the nodal lines for high frequencies.

Influence of the Horn

A horn as used with instruments for recording and reproducing sound is usually a conical or pyramidal tube, the smaller end of which is attached to the soundbox containing the diaphragm, while the larger end opens to the free air. The effect of the horn is to reinforce the vibrations which enter it due to the resonance properties of the body of air inclosed by the horn. The quantity and quality of resonance depends mainly upon the volume of the inclosed air and somewhat upon its shape. If the walls of the horn are smooth and rigid, they produce no appreciable effect upon the tone. But if the walls are rough or flexible, they may absorb or rapidly dissipate the energy of vibrations of

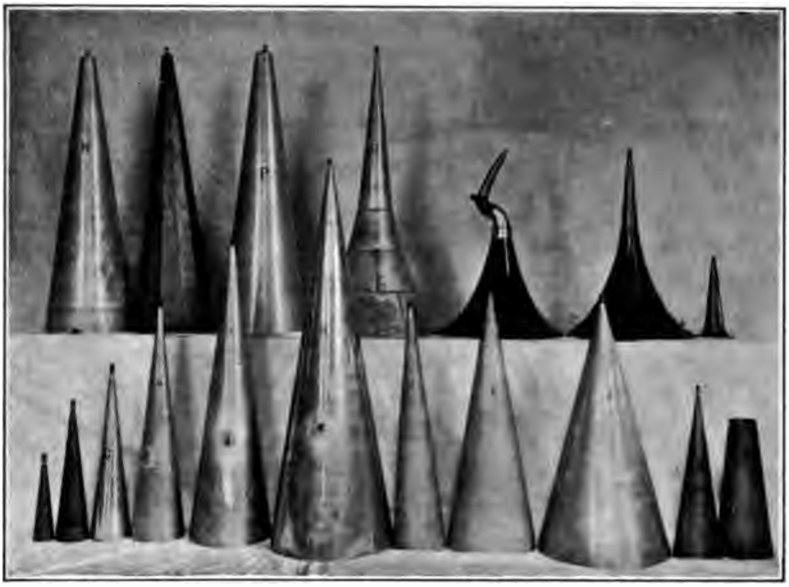


FIG. 122. Experimental horns of various materials, sizes, and shapes.

the air of certain frequencies and thus by subtraction have an influence upon tone quality. The horn of itself cannot originate any component tone, and hence cannot add anything to the composition of the sound. The horn is an air resonator and not a soundboard; any vibrations which the walls of the horn may have are relatively feeble and are received from the air which is already in vibration, while in the case of a soundboard, the air receives its vibration from the soundboard as a source. Because the horn operates through the inclosed air, it is a very sensitive resonator, and hence its usefulness when its action is understood and properly applied.

A horn used in connection with the diaphragm very greatly increases the response, but it also adds its own natural-period effects, which are quite complex. A variety of horns, shown in Fig. 122, were used in the experiments; these are of various materials, sheet zinc, copper, thick and thin wood, and artificial stone; one horn was made with double walls of thin metal, and the space between was filled with water. Probably the most rigid material, such as stone or thick metal, gives the best results. For convenience, however, sheet zinc is used with the phonodeik, and so long as the horn is supported under constant conditions, which are involved in the "correction curve" described later, this material is satisfactory.

A horn such as is used in these experiments has its own natural tones, which can be brought out by blowing with a mouthpiece as in a bugle; these tones are a fundamental with its complete series of overtones. The fundamental pitch can be heard by tapping the small opening with the palm of the hand.

When a horn is added to the diaphragm, the response is greatly altered; Fig. 123 shows the response curves for three horns of different lengths. In each curve the peaks corresponding to the fundamental of the horn, the octave, and the other overtones up to the seventh, are distinct. These peaks are indicated in the figure by H_1, H_2, etc., while the diaphragm peak is marked D.

In the upper curve, the peak due to the diaphragm comes between the peaks for the fundamental and the octave of the horn; the latter peaks are pushed apart, one being lowered in pitch and the other raised, so that the interval between them is two semitones more than an octave; the peak for the third partial is in its proper position. The horn reacts upon the diaphragm, causing it to have a period different from that which it had before the horn was applied.

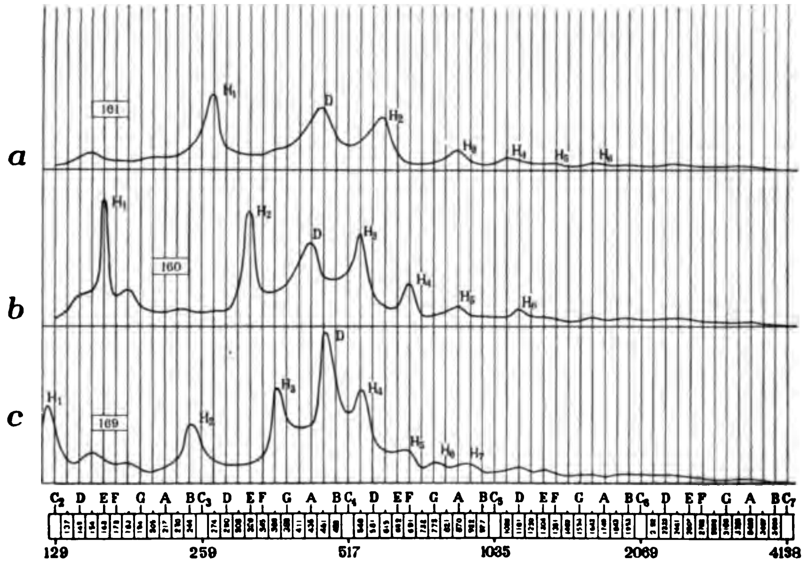


FIG. 123. Resonance peaks for horns of various lengths.

The middle curve shows the response with a longer horn; the diaphragm peak now comes between the peaks of the second and third partials, and both these and that of the fourth are displaced. The lower curve represents the response for a still longer horn.

A long horn seems to respond nearly as well to high tones as does a short one, while the response to low tones is much greater; the response below the fundamental of the horn is very feeble. The horn selected should be of such a length that its fundamental is lower than the lowest tone under investigation. For the study of vowel sounds, the horn employed has a length of about 48 inches, giving a fundamental frequency of about 125.

It is important that there are no holes, open joints, or leaks of any kind in the walls of the horn, because tones with a node in the position of a hole will be absent. A hole

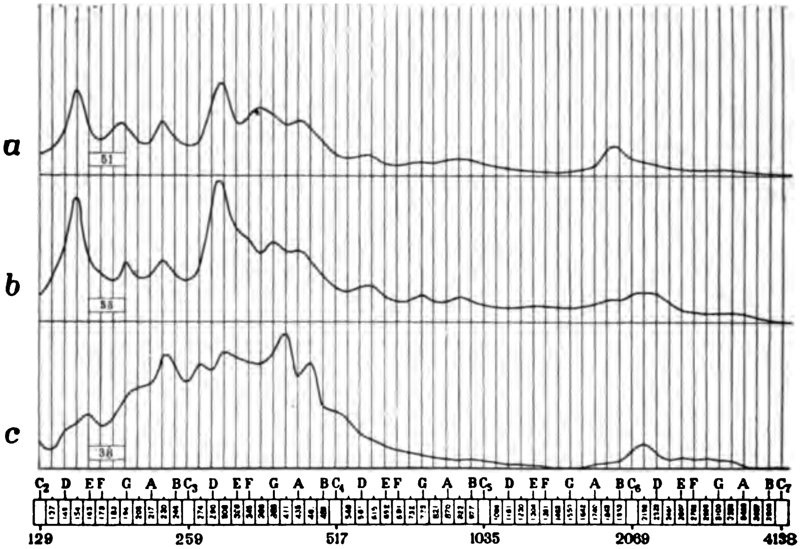


FIG. 124. Resonance peaks for horns of various flares.

one millimeter in diameter is sufficient to alter the response.

The flare of the horn has a great influence upon the response; Fig. 124 shows the responses obtained with three horns of the same length, but of different flares. The upper curve is the response for a narrow conical horn, the large end of which has a diameter equal to one fifth of the length; the middle curve is for a wide cone, the diameter of the open end being one half of the length; the lower curve is for a horn of flaring, bell shape. Widening the mouth increases the effect in a general way; the bell flare makes the natural periods indefinite, and heaps up the response near the fundamental, diminishing that for the higher tones.

The shape selected for use with the phonodeik is a cone

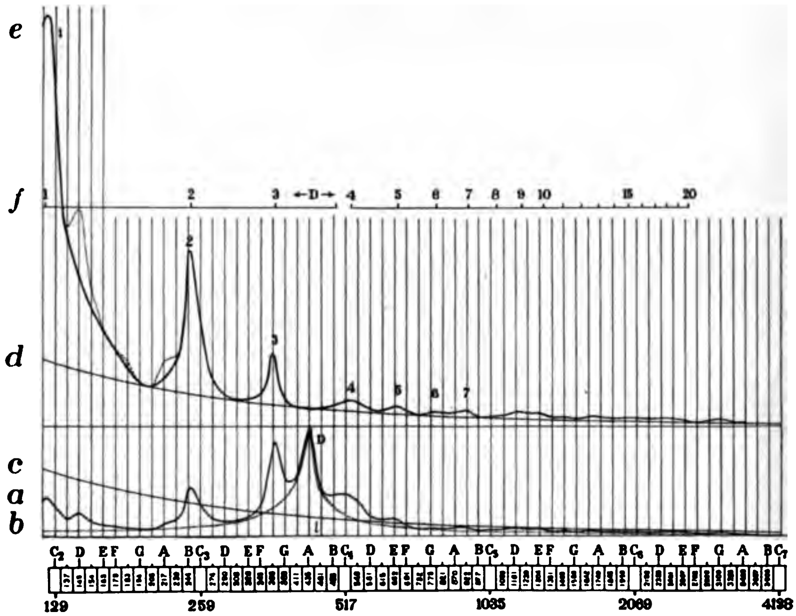


FIG. 125. Resonance effect of the horn.

of medium flare; this gives a good distribution of response, and the resonances are definite, but not too sharp to allow of correction.

The curve *a*, Fig. 125, is an actual response curve containing both horn and diaphragm effects, *b* is the diaphragm response, while *c* and *d* are the ideal curves previously explained. The effect of the natural period of the diaphragm is represented by the sharp peak *D*. If the diaphragm responded ideally, the curve *b* would coincide with *c* throughout its length, and *a* would then indicate the effects due to the horn alone. The ordinates of the curve *a* have been multiplied by such factors as will reduce the diaphragm curve to the ideal, and the results are plotted in *e*. The difference between the curve *e* and the ideal *d* is the effect due to the horn, corrected for the peculiarities of the diaphragm, in the manner explained later in this lecture.

The line *f* indicates the location of the natural harmonic overtones of the horn; the curve *e* shows by its peaks that the horn strongly reinforces the tones near its own fundamental, and, in a diminishing degree, those near all of its harmonics. The resonance of the horn increases the effects of all tones corresponding to the complete series of harmonics which the horn itself would give if it were blown as a bugle or hunting horn.

The diaphragm peak *D* comes between the peaks for the third and fourth partials of the horn, and it in effect divides the partials into two groups which are pushed apart in pitch; the amount of this displacement is shown by the gap in the line *f* near the fourth point.

Correcting Analyses of Sound Waves

The investigation of the effects of the horn and diaphragm of a sound-recording apparatus, a few details of which have been described, involved an unexpected amount of labor; it is estimated that the time required was equivalent to that of one investigator working eight hours for every working day in three and a half years. It has been shown that the horn and diaphragm introduce many distortions into the curves obtained with their aid, and that the distortions vary greatly with the conditions of the instrument. Unless these errors are eliminated and the true curves found, the records of the instrument will be without value because the incorrect and false curves can lead to no rational conclusions whatever. One may wonder, if the horn produces such disturbances, why it is not dispensed with in scientific research; the horn has been retained because the

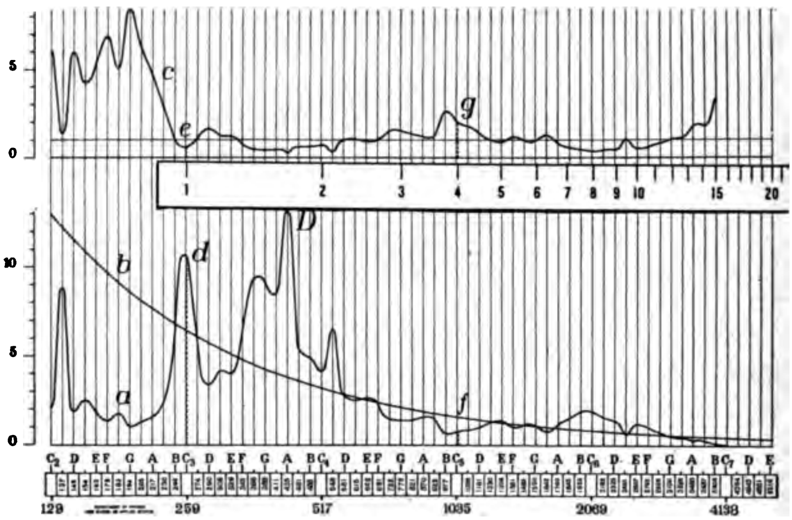


FIG. 126. Curves used in correcting analyses of sound waves.

sensitiveness of the recording apparatus is increased several thousand-fold by its use.

Whenever records are made for analytical purposes, the condition of the phonodeik is usually determined, as previously explained, by photographing its response to each of a set of sixty-one organ pipes of standard intensity, covering a range of frequencies from 129 to 4138. The actual response of the phonodeik in the form for research is shown by the irregular curve *a*, Fig. 126, while *b* is the desired or ideal response. The shape of the curve will vary with every change in the size or condition of the horn or diaphragm, with temperature changes, with the tightness of clamping of the diaphragm, with the nature of the room in which the apparatus is used, and with other conditions. The sharp resonance peak D is due to the free period of the diaphragm.

The ordinates of this curve give the amplitudes of the phonodeik records for sounds of various pitches, all being of the same intensity. The curve shows that the tone of any particular pitch, whether a single simple tone or a single component of a complex sound, in general produces a response either too large or too small as compared with the response due to other pitches. When any sound has been photographed with the phonodeik in the conditions here represented, and the amplitudes of all the components of the complex tone have been determined by analysis, it is necessary to correct each individual amplitude by multiplying it by a factor corresponding to its particular pitch. The factor for a tone of any pitch is the number by which the ordinate of the actual response curve for the given pitch must be multiplied to give the ordinate of the ideal curve.

These correction factors are obtained from a correction curve, c, Fig. 126, determined in the following manner. The sixty-one ordinates of the ideal curve b corresponding to semitones of the scale are divided by the corresponding ordinates of the response curve a; the quotients are the correction factors for these particular pitches. These factors are then plotted on the chart, and a correction curve is drawn through the points as shown at c. The correction curve is an inverse of the response curve, where one has a peak the other has a corresponding valley.

It is convenient to make a correction curve showing the factors for all pitches, since a number of photographs of sounds are often made with the phonodeik in the same condition, and the analyses of all are corrected from the same response curve.

The analysis of a curve having been recorded on a card, as explained on page 123, the correction factors, k, are measured from the chart of the correction curve, and are

CASE SCHOOL OF APPLIED SCIENCE — DEPARTMENT OF PHYSICS

ANALYSIS OF SOUND-WAVES

No. 1690, 1-10 — Source Organ Pipe — "Reedless Oboe" — Date April 23, 1915.

Tone C_3 — Abs. N 260 — Purpose Analysis

component, n	1	2	3	4	5	6	7	8	9	10
na_n	+ 22.6	+ 99.6	+ 101.1	+ 76.1	+ 44.5	− 49.1	+ 44.8	+ 24.9	− 11.5	− 7.1
nb_n	+ 94.1	− 87.2	− 42.5	− 7.5	− 26.1	− 10.6	− 3.8	− 66.7	− 36.9	− 21.7
nA_n	96.4	132.0	108.4	76.8	51.6	50.3	45.0	71.1	38.7	22.8
k_n, (1708)	0.6	0.7	1.5	2.1	0.9	1.0	0.6	0.4	0.5	0.6
nA_nk_n	57.9	92.4	164.1	161.3	46.4	50.3	27.0	28.4	19.4	13.8
$[nA_nk_n]^2$										
A_n	96.5	66.0	36.5	19.2	10.3	8.4	6.4	8.9	4.3	2.3
P_n	76.°	319.°	337.°	354.°	330.°	347.°	354.°	290.°	252.°	252.°
A_nk_n	57.9	46.2	54.7	40.3	9.3	8.4	3.9	3.6	2.2	1.4
Amplitude, %	25.4	20.3	24.0	17.7	4.1	3.7	1.7	1.6	1.0	0.6
Phase	0.°	167.°	109.°	50.°	310.°	250.°	182.°	41.°	288.°	211.°
Intensity, %										

Remarks — Sum — Analyzed R. F. H. — Synthesized

FIG. 127. Card form for the record of the analysis of a sound wave.

recorded on the line below the multiple amplitudes nA_n, Fig. 127. The harmonic scale (described on page 169) is placed on the chart with its first line on the pitch of the fundamental of the tone analyzed. The ordinates of the correction curve opposite the several harmonic points are the correction factors for the corresponding components; the ordinates are measured with a millimeter scale. For instance, the fundamental tone had the pitch 260, for which the phonodeik responded too much, as shown by the high peak at *d*; for this point the correction curve has an ordinate, *e*, equal to 0.6, which is the factor by which the amplitude of a tone of pitch 260, as recorded by the phonodeik, must be multiplied to reduce it to the ideal amplitude. For the fourth component, at point 4 on the scale, the response *f* is too small; the ordinate *g* of the correction curve is 2.1, the factor by which the response for this particular pitch, 1040, must be multiplied to make it equal to the ideal. A correction factor is obtained in this manner for each individual component.

The products, nA_nk_n, are found and each is divided by the order of its respective component, 1, 2, 3, etc., giving the corrected amplitudes, A_nk_n; these corrected amplitudes are presumed to be proportional to the actual amplitudes of the components of the original sound wave in air before it entered the horn of the recording apparatus. For comparison the corrected amplitudes are expressed as percentages of the sum of all the amplitudes, that is, the sum of all the component amplitudes is equal to 100.

Graphical Presentation of Sound Analyses

The object of the analysis of sound waves is the quantitative determination of the causes of tone quality. The analyses give directly the amplitudes and phases of the various components of a sound. The phases of the components which presumably have little effect upon the tone quality are not considered at the present time. Tone quality seems to depend only upon the relative intensities of the component tones. A simple comparison of the amplitudes of the various components gives an inadequate idea of the effects perceived by the ear; in fact, the relation between amplitude and loudness varies slightly for different ears and for different frequencies. The relative energies of the components, which may be derived from the observed and corrected amplitudes, while not corresponding exactly to the intensities as perceived by the ear, afford a close approximation, and these energies will be used in the present discussion to represent loudness.

If n is the order of a component tone in the natural se-

CASE SCHOOL OF APPLIED SCIENCE — DEPARTMENT OF PHYSICS

ANALYSIS OF SOUND-WAVES

No. 1690 — Source Organ Pipe — "Reedless Oboe." — Date April 23, 1915

1-10 — Tone C_3 — Abs. N 260 — Purpose Analysis

component, n	1	2	3	4	5	6	7	8	9	10
na_n	+ 22.6	+ 99.6	+ 101.1	+ 76.1	+ 44.5	− 49.1	+ 44.8	+ 24.9	− 11.5	− 7.1
nb_n	+ 84.1	− 87.2	− 42.5	− 7.5	− 26.1	− 10.6	− 3.8	− 66.7	− 36.9	− 21.7
nA_n	86.5	132.0	109.4	76.8	51.6	50.3	45.0	71.1	38.7	22.8
k_n (1708)	0.6	0.7	1.5	2.1	0.9	1.0	0.6	0.4	0.5	0.6
nA_nk_n	57.9	92.4	164.1	161.3	46.4	50.3	27.0	28.4	19.4	13.8
$[nA_nk_n]^2$	3352.	8538.	26829.	26018.	2153.	2530.	729.	807.	376.	190.
A_n	86.5	66.0	36.5	19.2	10.3	8.4	6.4	8.9	4.3	2.3
P_n	76.°	319.°	337.°	354.°	330.°	347.°	354.°	290.°	252.°	252.°
A_nk_n	57.9	46.2	54.7	40.3	8.3	8.4	3.9	3.6	2.2	1.4
Amplitude, %	25.4	20.3	24.0	17.7	4.1	3.7	1.7	1.6	1.0	0.6
Phase	0.°	167.°	109.°	60.°	310.°	250.°	182.°	41.°	288.°	211.°
Intensity, %	4.7	11.9	37.6	36.4	3.0	3.5	1.0	1.1	0.5	0.3

Remarks — Sum — Analyzed R. F. H. — Synthesized

FIG. 128. Card form for the record of the analysis of a sound wave.

ries, and A_nk_n its corrected amplitude, then $(nA_nk_n)^2$ is a number proportional to its energy or intensity, as was explained in Lecture II. As has been mentioned in this Lecture, the numbers nA_nk_n are found in the process of deriving the corrected amplitudes; the squares of these numbers are proportional to the energies of the several components. The record is completed by computing the percentage intensity of each component, Fig. 128, that is, the intensity of each component on the supposition that the loudness of the original complex sound as a whole is represented by 100.

The organ-pipe curve which has been used for the explanation of the method of analysis contains twelve components, as was proved by its synthetic reproduction, while many of the curves studied have twenty or more components. In the investigation of several thousand such curves there are hundreds of thousands of numerical operations, most of which are performed with the aid of calculating machines, adding machines, slide rules, and tables of squares and of products.

The final results of the analysis which are in the form of the relative intensities of the several partial tones, are most clearly presented when these intensities are plotted against a logarithmic scale of frequencies. A *logarithmic scale* of frequencies is one in which the spaces are proportional to the *ratios* of the frequencies; it is represented in actual sounds by the successive tones of a pianoforte. For instance, the successive octave points represent tones having frequencies in the ratio of 1 : 2 : 4 : 8 : 16, etc.; the ratio is the same throughout, and the actual distance in inches from any note to its octave on the scale, and also on the piano keyboard, is the same whether the note is in the lower part of the scale where the number of vibrations is small, or whether it is in the upper part of the scale having several thousand vibrations per second. The same is true of any other ratio, and hence the distance between any two specified partial tones is constant throughout the logarithmic scale, and is independent of the pitch of the fundamental.

For graphically presenting the analyses, a standard form of chart has been prepared, shown in Fig. 129, at the bottom of which is a logarithmic scale of frequencies. Equidistant vertical lines are drawn through points corresponding to the semitone intervals of the equally tempered musical scale in International Pitch from $C_2 = 129$ to $C_7 = 4138$; these lines may be compared to the strings of a piano from one octave below middle C to the highest note.

In addition to the prepared charts a bevel-edged harmonic scale is provided, with special rulings, so spaced as to correspond to the intervals of the natural series of partial tones, or harmonic tones, from 1 to 30. This scale, of course, will

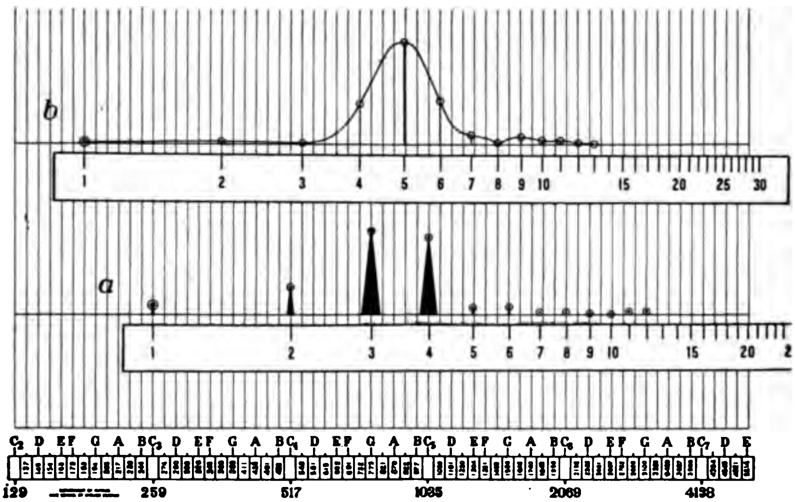


FIG. 129. Methods of diagraming the analyses of sound waves.

fit only a particular spacing; in the practical work two sizes of charts, and two sizes of harmonic scales, are used; in the larger the semitone interval is one centimeter and an octave measures 12 centimeters, while in the other the intervals are half this size.

The diagram of an analysis is made by placing the harmonic scale on the chart so that the first line corresponds to the actual pitch of the fundamental of the original sound; the other divisions of the scale then show by their locations the frequencies of the true harmonic overtones of this fundamental. An ordinate is measured through each harmonic point, equal in length, in millimeters, to the relative intensity, or percentage of intensity, of the corresponding partial. Since the total intensity of all the components is 100 per cent, the sum of all the ordinates on the diagram for any sound is 100 millimeters. The upper ends of the ordinates are sometimes represented by circles, the circle for the fundamental being larger than the others and having a black center.

Fig. 129, *a*, is the diagram of the analysis of the organ-pipe curve (Fig. 96) the data for which are given in the bottom line of the card shown in Fig. 128. The pitch of this tone is $C_3 = 259$, and the harmonic scale is placed on the chart to correspond. The loudness of each partial tone is the height of the corresponding circle above the base line; in this instance the third and fourth partials are the loudest. The pitch of each partial is shown by its position with respect to the scale at the bottom of the chart. The slender black triangles have no significance except to make the lengths of the ordinates more conspicuous.

In studying the analysis of sounds from certain sources, it is helpful to draw curves through the upper ends of the ordinates. Fig. 129, *b*, is such a diagram of the analysis of the vowel *a* in *father*, a photograph of which is given in Fig. 160, page 219.

The ordinates of these diagrams show the distribution of the energy in the sound with reference to its own harmonic partial tones which have definite pitches. A single analysis gives little or no information with regard to intermediate pitches, since the sound analyzed can have no intensity whatever for pitches other than those of its own partials.

A second analysis of a sound from a given source, as in voice analysis, intoned at a different pitch from that of the first, will have its partials at pitches intermediate between those already found. By comparing many analyses a curve can be drawn which shows the general distribution

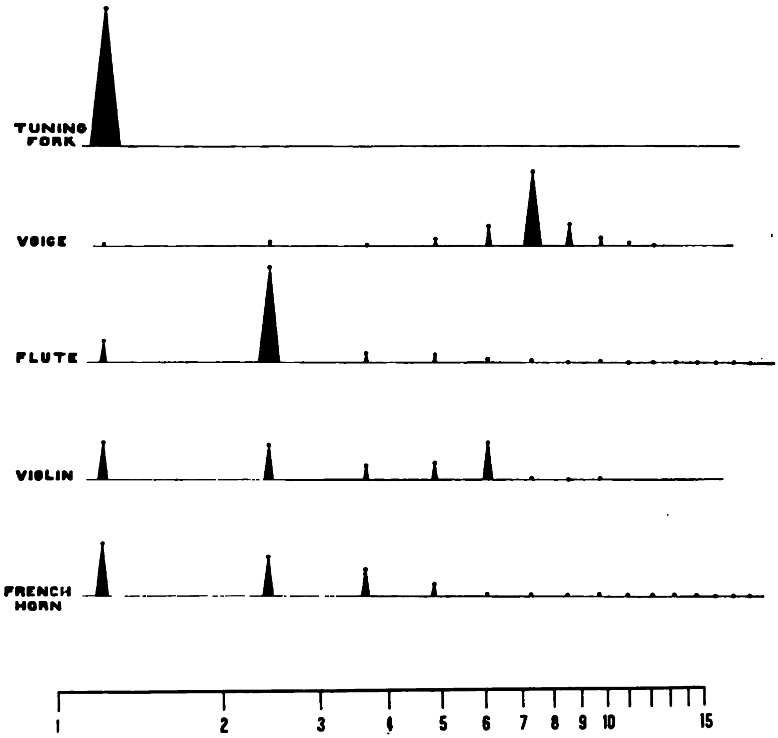


FIG. 130. Distribution of energy in sounds from various sources.

of energy from the source. The usefulness of such curves in connection with the study of vowel tones is more fully explained on pages 220 and 228.

These diagrams and curves showing the distribution of the energy in a sound, are not unlike the spectrum charts and emission curves obtained in the study of light sources. Corresponding to a monochromatic light we have what may be called the "mono-pitched" sounds of the tuning fork. This sound is simple, containing but a single component, and its diagram consists of one "strong line," as shown in Fig. 130. Other sources emit complex sounds, the energy being variously distributed among the several partials, as shown, for particular instances, in the figure. The horn gives the most uniformly distributed emission of sound energy, and its tone may be said to correspond acoustically to white light. The characteristics of instrumental and vocal tones are more fully discussed in the succeeding lectures.

Verification of the Method of Correction

As a test of the sufficiency of the method which has been developed for correcting analyses, one hundred and thirty photographs of tones from nine different instruments were made with four distinctly different combinations of horn and diaphragm, giving for each tone four sets of curves which are wholly unlike. A long horn was used with a large and a small diaphragm, and also a short horn with each diaphragm; the responses were such that the peaks in one instance corresponded in pitch with the valleys in another. Response and correction curves were made for each combination. After correction the various analyses of any one tone were identical.

Fig. 131 shows the photographs of the tone from an organ pipe made with three horn-and-diaphragm combinations; these would hardly be taken for records of the same sound. After analysis, the components for each curve were corrected for horn and diaphragm effects and then recompounded with the synthesizer, the three corrected curves

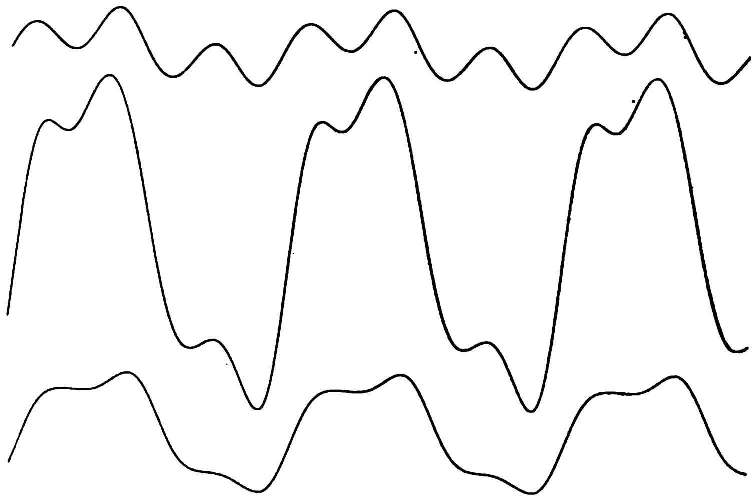


FIG. 131. Three curves for the same tone, made under different conditions.

being shown in Fig. 132. These curves show how successful the method is in reducing unlike curves for the same sound to practical identity.

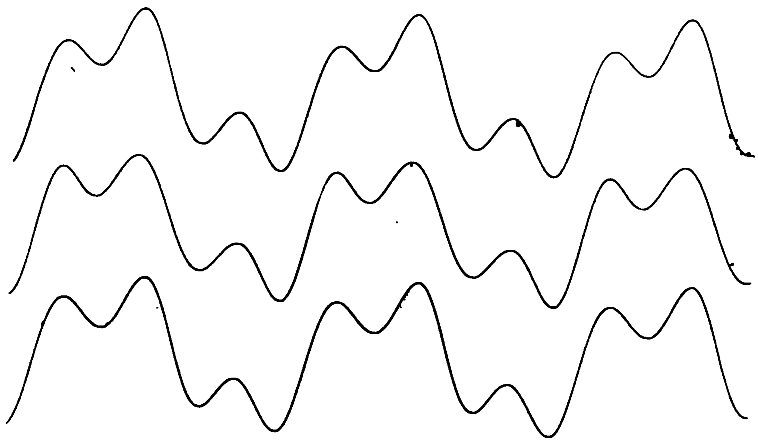


FIG. 132. The three curves of Fig. 131 corrected for instrumental effects.

Quantitative Analysis of Tone Quality

In Lectures III, IV, and V there has been described a quantitative method for the analytical study of tone quality. The method includes the arrangement of the working apparatus, and schemes for computing, reducing, presenting, and filing the results. The results so obtained are expressed in terms of the relative loudness of the various partial tones of an instrument. A determination of the relative loudness of the sounds of one instrument as compared with another is no doubt of much interest, but this is not included in the present discussion.

In the definitive study of a musical instrument or voice it is desirable that a large number of tones be photographed, perhaps four per octave, three semitones apart, throughout the whole compass; the tones should be sounded in three different intensities as *p*, *mf*, and *f*, or five intensities may be studied, by adding *pp* and *ff*; to eliminate errors, two or more combinations of horn and diaphragm may be used; the source of sound may be placed at various distances from the horn; response and correction curves must be taken before and after each change in the recording apparatus; such a study of one instrument may require a year's time for its completion. Any scheme less comprehensive than this will not give an adequate idea of the tone quality of a musical instrument.

LECTURE VI

TONE QUALITIES OF MUSICAL INSTRUMENTS

Generators and Resonators

Previous lectures have demonstrated that, in general, the sounds from musical instruments are composite, that is, all those which can be said to have characteristic quality are made up of a larger or smaller number of partial tones of various degrees of loudness. A scientific definition of the quality of a musical tone requires a statement of what particular partial tones enter into its composition and of the intensities and phase relations of these partials. In order to understand a musical instrument, we need to know how its tones are generated and controlled by the performer.

The sound producing parts of a musical instrument, in general, perform two distinct functions. Certain parts are designed for the production of musical vibrations. The vibrations in their original form may be almost inaudible, though vigorous, because they do not set up waves in the air, as is illustrated by the vibrations of the string of a violin without the body of the instrument; or the vibrations may produce a very undesirable tone quality because they are not properly controlled, as in the case of the reed of a clarinet without the body tube. Other parts of the instrument receive these vibrations, and by operation on a larger quantity of air and by selective control, cause the instrument to send out into the air the sounds which we ordinarily hear. These parts, which may be referred to as *generator* and *resonator*, are illustrated by the following combinations: a tuning-fork generator and its box resonator; the strings and soundboard of a piano; the reed and body tube of a clarinet; the mouth and body tube of an organ pipe; the vocal cords and mouth cavities of the voice. In the piano the soundboard acts as a universal resonator for all the tones emitted by the instrument; in the organ each pipe constitutes its own separate resonator; in the flute the body tube is adjusted to various different conditions by means of holes and keys, each condition serving for several tones.

The resonator cannot give out any tones except those received from the generator, and it may not give out all of these. The generator must therefore be capable of producing the components which we wish to hear, and these must in turn be emitted in the desired proportion by the resonator. If the generator produces partial tones which are undesirable, the resonator should be designed so that it will not reproduce them; if the generator produces tones which are of musical value but which the resonator does not reproduce, we do not hear them, and it is as though they were not produced at all. It follows that we can hear from a given instrument nothing except what is produced by the generator, and further we can hear nothing except what is also reproduced by the resonator; hence it may be that the most important part of an instrument is its resonator. The quality of any tone depends largely upon the kind and degree of sympathy, or resonance, which exists between the generator and the resonator.

Resonance

Every vibrating body has one or more natural periods in which it vibrates easily; to *tune* a sounding body is to adjust its natural period to a specified frequency. If a body capable of vibration is excited by any means whatever, and the exciting cause is removed, the body will usually vibrate freely in its natural frequency, or with its *free period.* If the exciting cause operates in this same frequency, the two are in *resonance*, that is, they are in tune; under these conditions the response of the body receiving the vibration is a maximum. If the exciting cause differs in frequency but slightly from that natural to the other body, there will still be response but in a lesser degree, that is, the resonance is not so *sharp.*[68] When the two bodies are quite out of tune, there will be very little resonance, and while the second body may still be made to vibrate, the response will be small. These conditions are well illustrated by the response curves described in the previous lecture.

When the resonator is out of tune with the generator, it is often made to vibrate with the generator, and it is then said to have *forced vibration.* In forced vibration, the two bodies have different natural frequencies, and the resulting forced frequency is in general not that natural to either body, each drawing the other more or less to a common intermediate frequency. In musical instruments usually the generator is much less influenced than is the resonator; for instance, a tuning fork in connection with a resonance box not exactly in tune, draws the air in the box to its own frequency much more easily than the air draws the fork. Koenig found that for a fork of 256 vibrations per second the maximum alteration of its frequency due to the drawing effect of the resonance box is produced when the box has a natural frequency of either 248 or 264. The fork in causing forced vibration of the air in the box draws the air from a frequency of 248 to 256.036 in the first case, and from 264 to 255.964 in the second; that is, the box being out of tune by 8 vibrations, the fork is forced out of its natural frequency by 0.036 vibration. When the box is out of tune by a musical semitone, the effect on the fork is less, being about 0.025 vibration.[69]

When a body is exactly in tune with the generator, that is, when it is in resonance, it may take up the vibrations with great ease and vigor; such a response is often called *sympathetic vibration.* This is often disagreeably illustrated by the rattle of bric-a-brac in a music room, or by the buzz of some part of the action of a piano or of a machine. Sympathetic vibration is demonstrated by means of two forks which are exactly in tune; if one fork is sounded loudly for a few seconds, the other fork is set in audible vibration, the only medium of communication being the air.

There are two distinct kinds of resonators. One kind having no definite vibration frequency of its own, responds to tones of any frequency and to combinations of these; it can reproduce all gradations of tone quality. A plate, such as the soundboard of a piano, is representative of this kind of resonator. The second type of resonator possesses a more or less definite natural frequency and, because of selective control, it reproduces sounds of particular quality only. Such a resonator will respond not only to tones corresponding to its fundamental, but also to tones in unison with its overtones. The second kind ot resonator is typified by the cylindrical brass box of the standard tuning fork described on page 51. This box has a very definite fundamental frequency and overtones which are high in pitch and not in tune with any overtone of the fork; therefore only the fundamental of the fork is reinforced, and the result is a pure simple tone.

A resonator does not create any sound; it can only take up the energy of vibration of the generator and give it out in a different loudness. It follows that for a given blow to a fork or a string, the more perfect the tuning of the resonator, the louder will be the sound and the shorter will be its duration. If the strings of two different pianos are struck with the same force of blow, that piano which gives the loudest sound will probably have the shortest duration of tone, while the one which begins the sound with moderate loudness will continue to sound longer or will "sing" better.

The loudness and the duration of the sound from an instrument are dependent upon the damping or absorption of the vibration in the instrument and its surroundings. The energy of the waves which travel outward from a sounding body is derived from the vibration of the body; usually not all of the energy of vibration is transferred, some being absorbed and transformed into heat through friction and the viscosity of the body. When the loss of energy is rapid, the amplitude of vibration decreases rapidly, and the vibrations are said to be *damped.*[70] These effects must be considered with resonance and consonance in the complete study of musical instruments.

Effects of Material on Sound Waves

Both the tones generated by a musical instrument and those reproduced, as well as those absorbed or damped, depend in a considerable degree upon the material of which the various parts of the instrument are constructed. While this fact is well known and commonly made use of in connection with certain classes of instruments, its truthfulness is often denied by the devotees of other instruments. The question of the influence of the material of which the body tube of a flute is made has not been settled after more than seventy years of widespread discussion. How does the tone from a gold or silver flute differ from that of a wooden flute? It was this specific question that suggested the investigations which, having passed much beyond the original inquiry, have furnished the material upon which this course of lectures is based.

FIG. 133. Organ pipes for demonstrating the influence of the walls on the tone.

The following experiments, suggested by those of Schafhäutl (Munich, 1879),[71] indicate the great changes in the tone of an organ pipe which may be produced by effects passing through the walls.[72] Three organ pipes are provided, as shown in Fig. 133. The first pipe, of the ordinary type used in physical experiments, is made of wood and sounds the tone $G_2 = 192$. Two pipes having exactly the same internal dimensions as the wooden one are made of sheet zinc about 0.5 millimeter thick. One of the zinc pipes has been placed inside a zinc casing to form a double-walled pipe, with spaces two centimeters wide between the walls; the outer wall is attached to the inner one only at the extreme bottom on three sides, and just above the upper lip-plate on the front side. These two pipes have exactly the same pitch, giving a tone a little flatter than F_2, which is more than two musical semitones lower than that of the wooden pipe of the same dimensions.

Using the single-walled zinc pipe one can produce the remarkable effect of choking the pipe till it actually squeals. When the pipe is blown in the ordinary manner, its sound has the usual tone quality. If the pipe is firmly grasped in both hands just above the mouth, it speaks a mixture of three clearly distinguished inharmonic partial tones, the ratios of which are approximately 1 : 2.06 : 2.66. The resulting unmusical sound is so unexpected that it is almost startling, the tone quality having changed from that of a flute to that of a tin horn.

Experiments with the double-walled pipe are perhaps more convincing. While the pipe is sounding continuously, the space between the walls is slowly filled with water at room temperature. The pipe, with the dimensions of a wooden pipe giving the tone G_2, when empty has the pitch F_2, and when the walls are filled with water the pitch is E_2; during the filling the pitch varies more than a semitone, first rising then falling. While the space is filling, the tone quality changes conspicuously thirty or forty times.

After the demonstration of these effects, one will surely admit that the quality of a wind-instrument may be affected by the material of its body tube to the comparatively small extent claimed by the player. The flute is perhaps especially susceptible to this influence because its metal tube is usually only 0.3 millimeter thick. It is conceivable that the presence or absence of a ferrule or of a support for a key might cause the appearance or disappearance of a partial tone, or put a harmonic partial slightly out of tune.

The traditional influence of different metals on the flute tone are consistent with the experimental results obtained from the organ pipe. Brass and German silver are usually hard, stiff, and thick, and have but little influence upon the air column, and the tone is said to be hard and trumpet-like. Silver is denser and softer, and adds to the mellowness of the tone. The much greater softness and density of gold adds still more to the soft massiveness of the walls, giving an effect like the organ pipe surrounded with water. Elaborate analyses of the tones from flutes of wood, glass, silver, and gold prove that the tone from the gold flute is mellower and richer, having a longer and louder series of partials, than flutes of other materials.

Mere massiveness of the walls does not fulfill the desired condition; a heavy tube, obtained from thick walls of brass, has such increased rigidity as to produce an undesirable result; the walls must be thin, soft, and flexible, and must be made massive by increasing the density of the material. The gold flute tube and the organ pipe surrounded with water, are, no doubt, similar to the long strings of the pianoforte, which have a rich quality; these strings are wound or loaded, making them massive, while the flexibility or "softness" is unimpaired. The organ pipe partly filled with water is like a string unequally loaded, its partials are out of tune and produce a grotesque tone. A flute tube having no tone holes or keys is influenced by the manner of holding; certain overtones are sometimes difficult to produce until the points of support of the tube in the hands have been altered.

Beat-Tones

When two simple tones are sounding simultaneously, in general, beats are produced, equal in number to the difference of the frequencies. When the beats are few per second, the separate pulsations are easily detected. When the beats are many, the ear does not perceive the separate pulses, and instead the sensation is that of a

Fig. 134. Photograph of beats produced by two tuning forks, giving the effect of a third tone, called a beat-tone.

third tone, which is as distinct and as musical as the two generating tones, and which has a frequency equal to the difference in the frequencies of the two generators; that is, its frequency is equal to the number of beats if such rapid beats could be heard. This tone is called a *beat-tone*.[73]

Fig. 134 is a photograph of the waves from two tuning forks having frequencies of $C_6 = 2048$ and $D_6 = 2304$, respectively, which are in the ratio of 8 : 9. If the two sets of waves are in like phase at a certain point, they combine and produce a curve of large amplitude, as at *a*, signifying a loud sound. This condition is repeated at regular intervals along the wave train, as at *b*, which is exactly 8 waves of one tone and 9 waves of the other from *a*.

A point *c*, midway between *a* and *b*, is 4 waves of one tone and 4½ waves of the other from *a*. When the motion at this point due to one wave is upward, that due to the other is downward, and the two neutralize each other, producing the effect shown in the curve and which corresponds to a minimum of sound. This neutralizing effect occurs regularly between each two reinforcements. The resultant sound of the two forks waxes and wanes as does the outline of the curve. When the number of fluctuations is less than 16 per second the ear hears the separate pulsations as beats; when the number of pulsations is large, the effect upon the ear is that of a continuous simple tone of a frequency equal to the number of beats per second; this effect is the beat-tone. There are 256 beats per second in the instance described, and the ear hears not only the two real fork-tones, $C_6 = 2048$ and $D_6 = 2304$, but also a third beat-tone, of the pitch $C_3 = 256$. The latter sounds just as real as the other two tones, but it has no physical existence as a tone; there is no vibrating component of motion corresponding to the beat-tone, an analysis of the wave form showing only the two components due to the forks. While beat-tones are purely subjective, yet they affect the ear as do real tones. These subjective partials have great influence on the tone quality of many instrumental and vocal sounds as perceived by the ear. This influence has never been fully appreciated.

Identification of Instrumental Tones

There are many who, listening to a full orchestra, are able to distinguish the tones of a single instrument even when all the instruments are being played. We usually think this possibility is dependent on the characteristic quality of the instrument, but investigation indicates that tone quality is only one of several perhaps equally important factors of identification. Other aids in the differentiation are the attendant and characteristic noises of the instrument, such as the scratching of the bow, the hissing of the breath, and the snapping of the plucked strings. A further very important help to the observer, especially if he is not a trained musician, is the visual observation of the motions of the performer; the synchronism of these movements with the changes of the melody calls attention to the particular instrument. Every one attending a concert desires to see the musicians as well as to hear them; a seat in a concert hall which allows no view of the musicians is considered most undesirable.

It is very difficult to keep the many instruments of an orchestra in perfect tune; indeed it is almost certain that perfect tuning is unattainable. The imperfections of tuning prevent the harmonious blending of the sounds of the various instruments, and an individual instrument may be separated from the mass of sound by its particular pitch, which condition will help to differentiate it and assist the hearer in the identification. The hearer may not be conscious of the lack of tuning, and, indeed, many persons are not over-critical in this respect.

Helmholtz says[74] that the proper musical qualities of the tone from a fork and of that produced by blowing across the mouth of a bottle, both being simple, are identical. Certain tones of the flute are also simple, and therefore of the same quality as those of the tuning fork. Experimental demonstration has proved that when the auditor is removed from the sounding bodies so that he is unable either to see them or to hear the attendant noises, he cannot tell whether it is the fork, the bottle, or the flute, that produces the tone. It is a fact that certain tones can be produced on the flute, in the lower register, which cannot be distinguished by the trained musician from certain tones of the violin, and like similarities are possible with pairs of other instruments. Of course, any instrument can probably produce certain peculiar tones which are impossible of imitation by any other instrument.

The musician and the scientist are interested in the distinguishing features of the tone qualities of the various orchestral instruments and of other sources of musical sounds. The true characteristic tone of an instrument is the sustained and continuable sound produced after the sound has been started and has reached what may be called the steady state; this steady sound is usually free from the noises of generation. Systematic analyses covering the entire scale in various degrees of loudness have been made for the flute, violin, horn, and voice, and less complete analyses have been made for other instruments; these studies are to be continued until a general survey has been made of the entire tonal facilities of instrumental music. The analyses which have been completed make it possible to describe the distinguishing characteristics of the tones of the several instruments.

The Tuning Fork

The musical instrument which gives the simplest and purest tone, as mentioned in Lecture II, is a tuning fork in connection with a resonator. A photograph of the tone from a fork sounding middle C = 256 is shown in Fig. 135;

such a tone needs no other description than the statement that it is simple. This wave form will be recognized as that produced by the simplest possible vibratory motion,

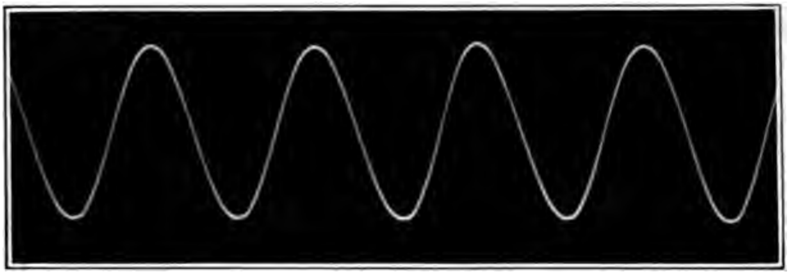


FIG. 135. Photograph of the simple tone from a tuning fork.

simple harmonic motion. For comparison the analysis of such a simple tone is shown on the diagram of various tones in Fig. 130, page 171; since the tone has but one component the diagram consists of one line only.

FIG. 136. Photograph of the clang-tone from a tuning fork.

If a fork is struck a sharp blow with a wooden mallet or other hard body, it can be made to give a ringing sound in which the ear easily distinguishes a high-pitched clang-tone in addition to the fundamental heard when the fork is sounded with a soft hammer; this clang-tone is the first natural overtone of the fork. Fig. 136 is a photograph of the tone from a fork struck with a wooden mallet, the kinks in the wave form being due to the overtone thus produced. Inspection shows that the relation of the small wave to the large one occurring at the point *a* does not recur till

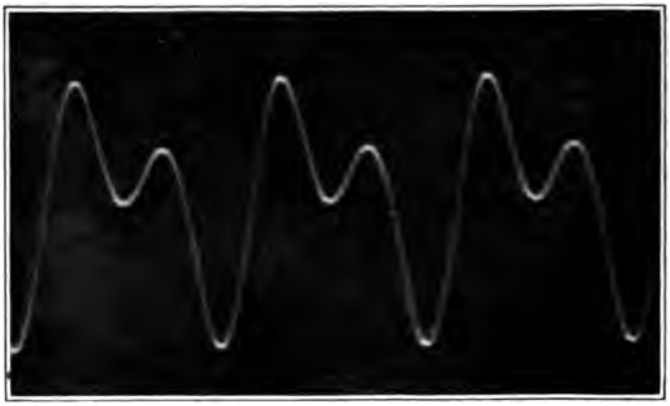


FIG. 137. Photograph of the tone of a tuning fork having the octave overtone.

the fourth succeeding wave, at *b*; in the four large waves there are twenty-five kinks due to the small one, that is, the frequency of the overtone is about 6.25 times that of the fundamental. Since there is not an integral number of the smaller waves to one of the larger, the partial is inharmonic or out of tune, and hence the sound is clanging or metallic rather than musical.

When a tuning fork mounted on a resonance box is sounded by vigorous bowing, it sometimes produces a strong octave overtone; such a tone is not natural to either the fork or the box, and is probably due to some peculiar condition of the combination which has not yet been fully explained.[75] A photograph of this unusual tone from a fork is shown in Fig. 137.

Tuning forks have been used as musical instruments in connection with keyboards like those of the piano or organ. The tones are remarkably sweet and of greater purity than those obtainable from any other instrument; but the very fact of purity, that is, the absence of higher partial tones, renders the music monotonous and uninteresting, and such devices have not survived the experimental stages.

The Choralcelo, an instrument of recent design,[76] produces a sustained tone, having the same general characteristics as that of the tuning fork. The vibrations are produced by electromagnets, through which flow interrupted, direct, electric currents, the pulsations of which are of the same periods as those of the bodies to be set in motion. The sources of sound may be piano strings or ribbons of steel drawn over a soundboard, which are set in vibration by the direct action of the magnets. In other instances, bars of wood, aluminum, or steel are used in connection with resonators, or diaphragms of special construction are fastened to the ends of resonant tubes; soft iron armatures are attached to the bars and diaphragms, which are set in vibration by the pulsations of the magnets, and thus the air in the resonators is moved and the tones are produced. The tones so obtained are nearly simple in quality, consisting mainly of a fundamental. The overtones, naturally absent, are provided by sounding corresponding generators in accordance with a scheme of tone combinations which can be carried out conveniently by means of stops or controllers operating switches in the electrical apparatus. The Choralcelo produces tones which are very clear and vibrant and of great carrying

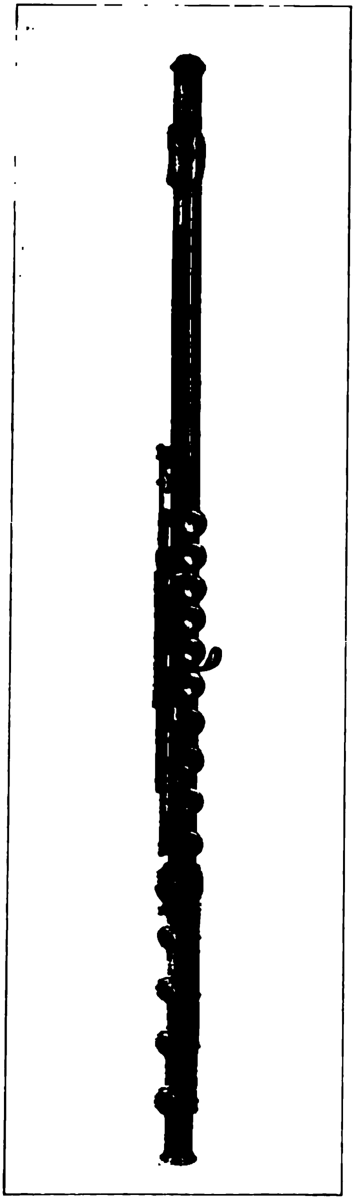


FIG. 138. The flute.

power, due, perhaps, to the strong fundamental component. The combinations of such sounds produce unique tonal effects, of remarkable musical quality, and the possibilities of synthetic tone development are great.

THE FLUTE

The flute in principle is of the utmost simplicity; it consists of a cylindrical air column a few inches in length, set into longitudinal vibration by blowing across a hole near the end of the tube which incloses the air column. The holes in the body of the flute, with the keys and mechanism, Fig. 138, serve only to control the effective length of the vibrating air column.

While the flute is simple acoustically, the manipulation of the instrument in accordance with the requirements of music of the present time, requires a key-mechanism of considerable complexity and of the finest workmanship. The flute has been developed to an acoustical and mechanical perfection perhaps not attained by any other orchestral instrument. This is largely due to the artistic and scientific studies of the instrument made by Theobald Boehm, of Munich, who devised the modern system of fingering in 1832, and invented the cylindrical-bore, metal tube, with large covered finger holes, in 1847.[77]

The flute gives the simplest sound of any orchestral instrument, and this is especially true when it is played softly. The paucity of overtones causes its sound to blend more readily with that of other instruments or the voice, and prevents the poignant expressiveness of the stringed and reed instruments; nevertheless, the flute has an expression peculiar to itself, and an aptitude for rendering certain sentiments not possessed by any other instrument. Berlioz says: "If it were required to give a sad air an accent of desolation and of humility and resignation at the same time, the feeble sounds of the flute's medium register would certainly produce the desired effect." The flute, because of its agility and ability to play detached and extended passages, arpeggios, and iterated notes, as well as because of its light tone quality, is suited to music of the gayest character. The flute tone is often described as sweet and tender; Sidney Lanier, himself an accomplished flutist, describes this tone in "The Symphony": [78]

> "But presently
> A velvet flute-note fell down pleasantly
> Upon the bosom of that harmony,
> And sailed and sailed incessantly,
> As if a petal from a wild-rose blown
> Had fluttered down upon that pool of tone
> And boatwise dropped o' the convex side
> And floated down the glassy tide
> And clarified and glorified
> The solemn spaces where the shadows bide."

Sound waves from the flute are shown in Fig. 139, in which are three curves for the same tone, $G_3 = 388$, played *p*, *mf*, and *f*; when played softly the tone is nearly simple, an increase in loudness adds first the octave, and then still higher partials.

About a thousand photographs of flute tones have been analyzed, including every note in the scale of the instrument, each in several degrees of loudness; flutes made of various materials have been studied, as wood, silver, gold,

FIG. 139. Three photographs of the tone of a flute, played *p*, *mf*, and *f*.

and glass, and the effects of different sized holes have been investigated. Some of the results of the analyses of the tones of the gold flute will be described; flutes of other materials have the same general characteristics, except that the overtones are fewer and weaker.

The average composition of all the tones of the low register of the flute, one octave in range, when played *pianissimo*, is shown by the lower line of Fig. 140; these tones are nearly simple, containing about 95 per cent of fundamental, with a very weak octave and just a trace of some of the higher partials. The *pianissimo* tones of the middle register, shown on the second line of the figure, are simple, without overtones.

When the lower register is played *forte*, it is in effect overblown, and the first overtone becomes the most prominent partial, as shown on the third line; the fundamental is weak,

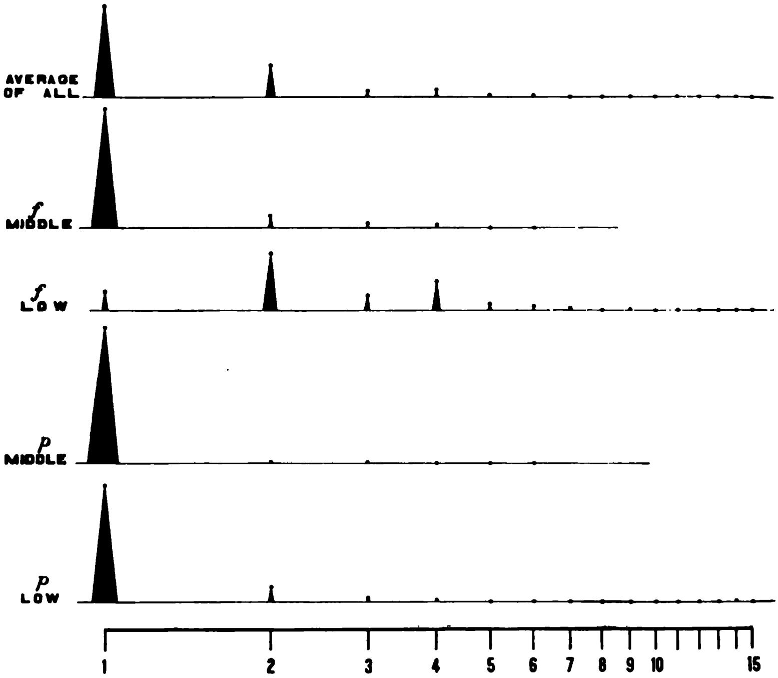


FIG. 140. Analyses of flute tones.

being just loud enough to characterize the pitch. The player is often conscious of the skill required to prevent the total disappearance of the fundamental and the passing of the tone into the octave. The tones of the low register, when played loudly, have as many as six or eight partials, and at times these sounds suggest the string quality of tone.

The tones of the middle register played *forte* consist mainly of fundamental with traces of the second and third partials. In this respect these flute tones are very similar to those of the soprano voice of like pitch, a fact which is made use of in the duet of the Mad Scene in the opera "Lucia di Lammermoor."

The average of all of the tones of the lower and middle registers of the flute, shown in the upper line of the figure, leads to the conclusion that the tone of the flute is characterized by few overtones, with the octave partial predominating.

The tones of the highest register have been analyzed and found to be practically simple tones. This result is to be expected from the conditions of tone production for the higher tones; the air column is of a diameter relatively large as compared with its length, and it is difficult to produce loud overtones in such an air column.

The Violin

The strings of a violin, Fig. 141, are caused to vibrate by the action of the bow, and these vibrations are transmitted through the bridge and body of the instrument to the air; not only does the body affect the air by its surface movements, but the interior space acts as a resonance chamber.

Helmholtz has made a study of the vibrating violin string, and has developed the mathematical equations defining the motion; [79] Professor H. N. Davis has investigated the longitudinal vibrations of strings in a manner to throw much light upon the subject; [80] Professor E. H. Barton and his colleagues have photographed the movements of the string and body of the instrument; [81] and P. H. Edwards and

C. W. Hewlett have studied the tones of violins of differing quality.[82]

By means of the vibration microscope Helmholtz observed the vibrations of the string and plotted the form of its movements, point by point, as shown in Fig. 142. A photograph of a sound wave from a violin is given in Fig. 143; the form is, in general, identical with the Helmholtz diagram; this identity is remarkable when it is remembered that the photograph is the wave in air, from the body of the instrument, while the diagram represents the movements of the string. The photograph shows what may be considered the typical form of a violin wave, but it is not the common form; this particular shape depends upon a critical relation between the pressure, grip, and speed of the bow, and upon the place of bowing and the pitch of the tone. The usual variations in bowing disturb the regularity of the vibrations, and produce a continually changing wave form. This is an

FIG. 141. The violin.

indication of the fact that a great variety of tone quality can be produced by the usual changes in bowing. Berlioz, in describing the orchestral usefulness of the violin, says: "From them is evolved the greatest power of expression,

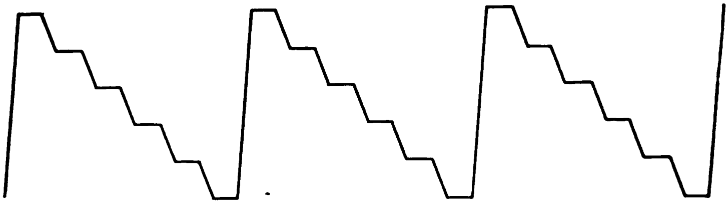


FIG. 142. Helmholtz's diagram of the vibrations of a violin string.

and an incontestable variety of qualities of tone. Violins particularly are capable of a host of apparently inconsistent shades of expression. They possess as a whole force, lightness, grace, accents both gloomy and gay, thought, and

FIG. 143. Photograph of the tone of a violin.

passion. The only point is to know how to make them speak."

The tone quality, as well as the wave form, remains constant so long as the bowing is constant in pressure, speed, and direction. The direction of bowing may be skillfully reversed without changing the tone quality. Fig. 144 is a photograph of the wave form when a change of bowing occurs; the first part of the curve is for an up-bow, while the other part is produced by the down-bow; the curve is symmetrically turned over with every change in the direction of bowing, while the confusion caused by the change produces a noise which lasts about two hundredths of a second. Analytically, the turning over of the curve means that the phases of all the components are reversed; the ear does not detect any change in tone quality due to the

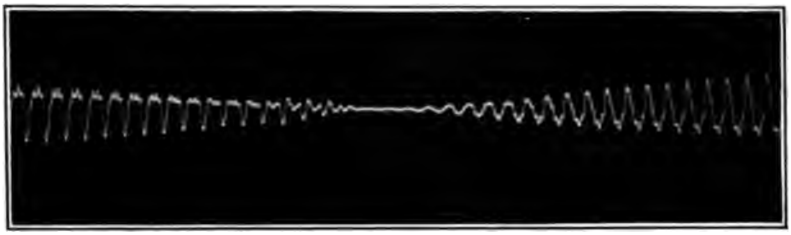


FIG. 144. Photograph of the tone of a violin at the time of reversal of the bowing.

reversal of phases, and this fact supports the statement that tone quality is independent of phase.

Photographs were taken of a series of tones on each string of the violin, of three degrees of loudness. The average results of the analysis of loud tones from the four strings are shown in Fig. 145. For the lower sounds the fundamental is weak, as indeed it must be, since these tones are lower than the fundamental resonance of the body of the violin; the tones from the three higher strings have strong fundamentals. The ear perceives a fundamental in the lower tones of the violin, and this must result from a beat-tone produced by adjacent higher partials which are strong. The tones from the three lower strings seem to be characterized by strong partials as high as the fifth, while the E string gives a strong third. In general the tone of the violin is characterized by the prominence of the third, fourth, and fifth partials; and while the violin generates a larger series of partials than does the flute, yet it is not equal to the brass and reed instruments in this respect. The great advantage of the violin over

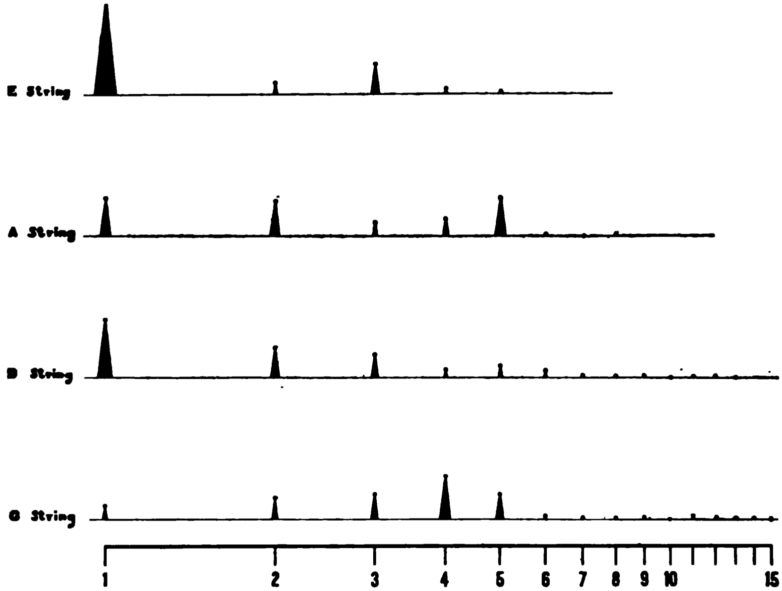


FIG. 145. Analyses of violin tones.

all other orchestral instruments in expressiveness is due to the control which the performer has over the tone production.

THE CLARINET AND THE OBOE

The study of reed instruments has not been completed, but the analyses of many individual tones show interesting characteristics. The clarinet, Fig. 146, generates sound by means of a single reed of bamboo which vibrates against the opening in the mouthpiece; these vibrations are controlled and imparted to the air by the body tube. The body has a uniform cylindrical bore, at the lower end of

FIG. 146. The clarinet.

FIG. 147. The oboe.

which is a short, bell-shaped enlargement. The keys vary the resonance of the interior column of air and thus control the pitch.

The oboe, Fig. 147, has a mouthpiece consisting of two reeds which vibrate against each other. The body with its keys forms a resonance chamber of various pitches, but it differs from the clarinet in that the bore is conical throughout.

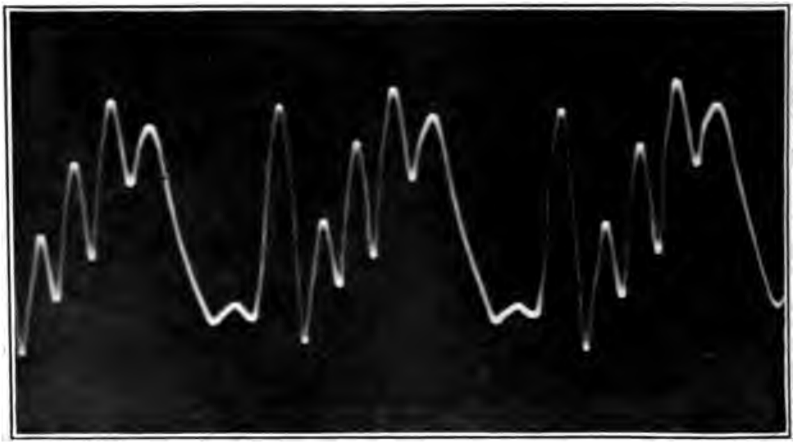


FIG. 148. Photograph of the tone of an oboe.

The photograph of the tone from an oboe, Fig. 148, and that from a clarinet, Fig. 149, both show deep kinks in the wave form; these kinks indicate the presence of relatively

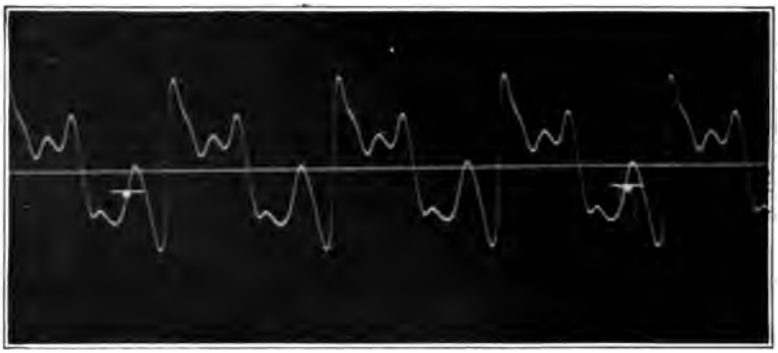


FIG. 149. Photograph of the tone of a clarinet.

very loud higher partials which, no doubt, produce the reedy tone quality of these instruments. The presence of beats, that is, the recurrence in each wave length of portions where the kinks are neutralized, shows that there are two adjacent high partials of nearly equal strength, as was explained under Analysis by Inspection in Lecture IV. The average of several analyses, Fig. 150, shows that the oboe tone has twelve or more partials, the fourth and fifth predominating, with 30 and 36 per cent respectively of the total loudness. The clarinet tone may have twenty or more partials; the average of several analyses shows twelve of importance, with the seventh, eighth, ninth, and tenth predominating; the seventh partial contains 8 per cent of

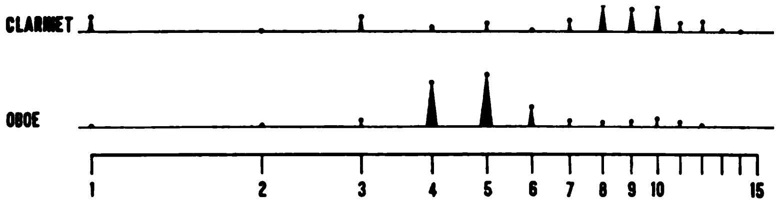


FIG. 150. Analyses of the tones of the oboe and the clarinet.

the total loudness, while the eighth, ninth, and tenth contain 18, 15, and 18 per cent respectively.

The statement is often made that the seventh and ninth partials are "inharmonic" and that their presence renders a musical sound disagreeable. The seventh and ninth partials are just as *natural* as any others; a partial is not inharmonic because it is the seventh or ninth in the series of natural tones; any partial whose frequency is an exact multiple of that of the fundamental is truly harmonic; a partial is inharmonic when it is not an exact multiple of the fundamental frequency, whether it is the second or ninth, or any other of the natural series. If the wave form of a sound is periodic, its partials must all be harmonic, and such a sound is musical. The clarinet gives periodic waves, which, as the analysis shows, contain loud seventh and ninth partials; these partials may almost be said to be the characteristic of the tone, but they are in tune, and are harmonic, and the clarinet tone has a very beautiful musical quality. Lanier in "The Symphony" says:[78]

> "The silence breeds
> A little breeze among the reeds
> That seems to blow by sea-marsh weeds;
> Then from the gentle stir and fret
> Sings out the melting clarionet."

The adjective "melting" seems to the author not merely a poetic term, but a real description of the clarinet as heard in the orchestra.

The Horn

The horn, Fig. 151, is a brass instrument of extreme simplicity, consisting of a slender conical tube, sometimes more than eighteen feet long, with a conical cup-shaped mouthpiece, and a large flaring bell. In its typical form there are no apertures in the walls of the tube, and no valves; but the modern horn usually has valves, as shown in the figure.

The tone of the horn is described by Lavignac as "by turns heroic or rustic, savage or exquisitely poetic; and it is perhaps in the expression of tenderness and emotion that it best develops its mysterious qualities." The scientific analysis shows causes for the variety of musical effects, for the horn produces tones of widely differing composition from one as soft and smooth as a delicate flute tone to a "split" tone that is tonally disrupted by strong higher partials. The low sounds of the horn are rich in overtones, containing the largest number of partials yet found in any musical tone. The analysis of the wave for the tone

$D_2 = 162$, Fig. 152, shows the presence of the entire series of partials up to thirty, with those from the second to the six-

FIG. 151. The horn.

teenth about equally loud; a diagram of this analysis is given on the lower line of Fig. 153.

The results of analyses of various other tones from the horn are also given in Fig. 153. The second line shows the

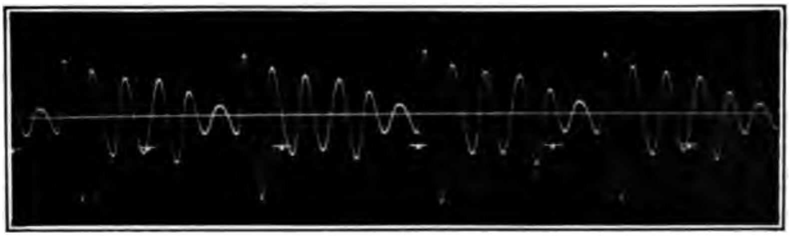


FIG. 152. Photograph of the tone of a horn.

average composition of loud, medium, and soft tones ranging over the entire compass, indicating a strong fundamental followed by a complete series of partials, more than twenty in number, of gradually diminishing intensity. The tone quality thus indicated approaches more nearly to the ideal described on page 211 than does that of any other instrument so far investigated.

A "chord tone" which seems to be made by humming with the vocal chords while playing, shown in the third line of the figure, has the fourth, fifth, and sixth partials the most prominent, which give the common chord, *do*, *mi*, *sol*.

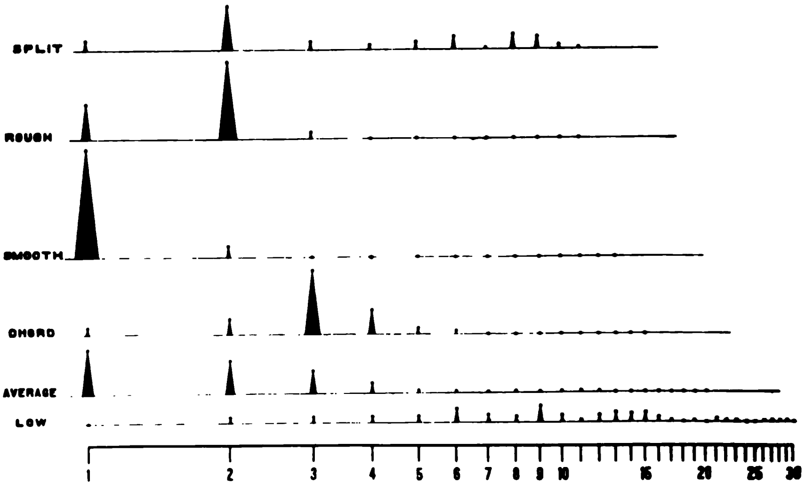


FIG. 153. Analyses of tones of the horn.

A "smooth" tone is produced when the player muffles the tone more or less by putting his hand in the bell of the horn. The analysis of such a tone, shown in the fourth line, indicates that it is nearly simple and is very much like the lower tones of the flute when played softly.

The "rough" tone is played more loudly and without muffling by the hand; the analysis, line five, shows an octave overtone which is louder than the fundamental, and a weak third partial.

Another quality of tone, shown on the top line, is called a "split tone"; this tone is literally split into many partials and distributed uniformly from the fundamental to the twelfth.

The Voice

The sounds of the voice originate in the vibrations of the vocal cords in the larynx, the pitch being controlled largely by muscular tension, while the quality is dependent mostly upon the resonance effects of the vocal cavities.

The tones of the singing voice have not been analyzed except in connection with the vowels, the results of which are described at length in Lectures VII and VIII. Fig. 154 shows the curve for a bass voice (E C) intoning the vowel *a* in *father* on the note $F_1\sharp$, = 92, . The loud partials in this tone are evidently of a high order, since

Fig. 154. Photograph of a bass voice.

there are many large kinks in one wave length. A diagram of the analysis of the curve is given in the lower line of Fig. 156; it shows that the seventh, eighth, ninth, and eleventh partials are the strongest.

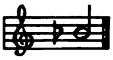
The voice E E M intoned the same vowel on the soprano pitch of $B_3\flat$, , producing the curve shown in Fig. 155, the analysis of which is given in the upper line of

Fig. 156. This curve is simple, and is seen at a glance to contain but one strong partial, the second overtone or octave.

These two voices, and their corresponding curves, are very unlike, yet the ear recognizes the same vowel from both. The vowel characteristics of the bass voice, represented by the seventh, eighth, ninth, and eleventh partials, are accompanied by six lower partials, the first of which determines the pitch while the others give the bass quality of the individual voice. The vowel characteristic for the other voice is the second partial, the pitch is determined by the first or fundamental, while a series of five or more higher partials produce the individuality of the soprano tone.

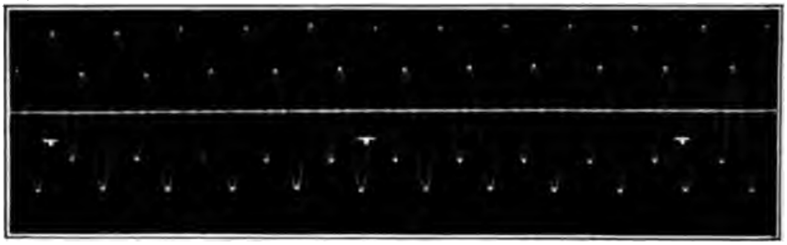


FIG. 155. Photograph of a soprano voice.

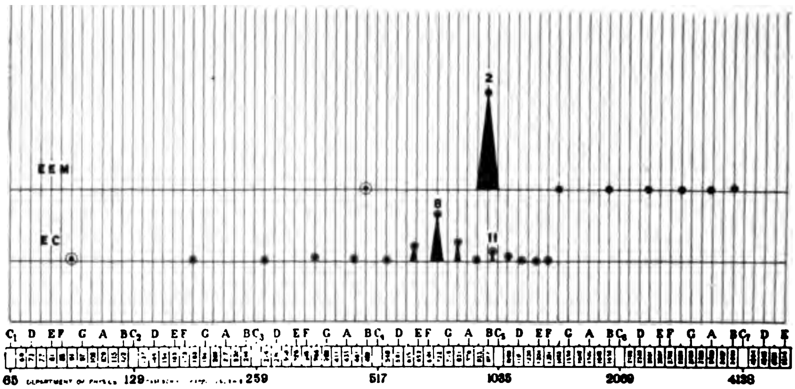


FIG. 156. Analyses of tones of bass and soprano voices.

TONE QUALITIES OF MUSICAL INSTRUMENTS

THE PIANO

The vibrations in a piano originate in the strings which are struck with the felt-covered hammers of the key action, while the sound comes mostly from the soundboard. The relations of strings and soundboard have been considered under Resonators and Resonance in the beginning of this lecture.

The lower tones of the piano are found to be very weak in fundamental, but to have many overtones, partials as high as the forty-second having been identified. These high partials are loud enough to be heard by the unaided ear after attention has been directed to them. These characteristics are entirely consistent with the nature of the source, which is a slender metal string struck with a hammer.

The higher tones of the piano, originating in much shorter strings under high tension, have few partials, and the loudest component is often the second partial or octave. The tones from the middle portion of the scale contain ten or more partials of well distributed intensity.

The piano is perhaps the most expressive instrument, and therefore the most musical, upon which one person can play, and hence it is rightly the most popular instrument. The piano can produce wonderful varieties of tone color in chords and groups of notes, and its music is full, rich, and varied. The sounds from any one key are also susceptible of much variation through the nature of the stroke on the key. So skillful does the accomplished performer become in producing variety of tone quality in piano music, which expresses his musical moods, that it is often said that something of the personality of the player is transmitted by the "touch" to the tone produced, something which is quite independent of the loudness of the tone. It is also claimed that a variety of tone qualities may be obtained from one key, by a variation in the artistic or emotional touch of the finger upon the key, even when the different touches all produce sounds of the same loudness. This opinion is almost universal among artistic musicians, and doubtless honestly so. These musicians do in truth produce marvelous tone qualities under the direction of their artistic emotions, but they are primarily conscious of their personal feelings and efforts, and seldom thoroughly analyze the principles of physics involved in the complicated mechanical operations of tone production in the piano. Having investigated this question with ample facilities, we are compelled by the definite results to say that, if tones of the same loudness are produced by striking a single key of a piano with a variety of touches, the tones are always and necessarily of identical quality; or, in other words, a variation of artistic touch cannot produce a variation in tone quality from one key, if the resulting tones are all of the same loudness. From this principle it follows that any tone quality which can be produced by hand playing can be identically reproduced by machine playing, it being necessary only that the various keys be struck automatically so as to produce the same loudness as was obtained by the hand, and be struck in the same time relation to one another. There are factors involved in the time relations of beginning the several tones of a chord or combination, which are not often taken into account; a brief notice of the nature of piano tone will enable us to establish this conclusion.

Two photographs of piano tones are shown, the first, Fig. 157, being of the note one octave above middle C and the other, Fig. 158, of the note one octave below. The first photograph shows two important features: the sound rises to its maximum intensity in about three one-

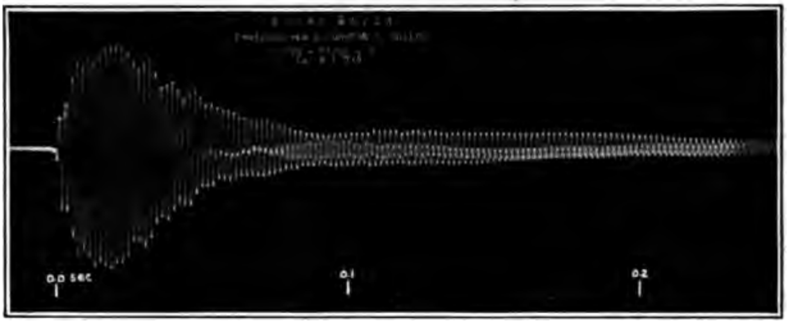


Fig. 157. Photograph of the tone of a piano.

hundredths of a second, and in one fifth of a second it has fallen to less than a tenth of its greatest loudness; it then gradually dies out, but with a progressive change in quality. In the beginning the fundamental is the

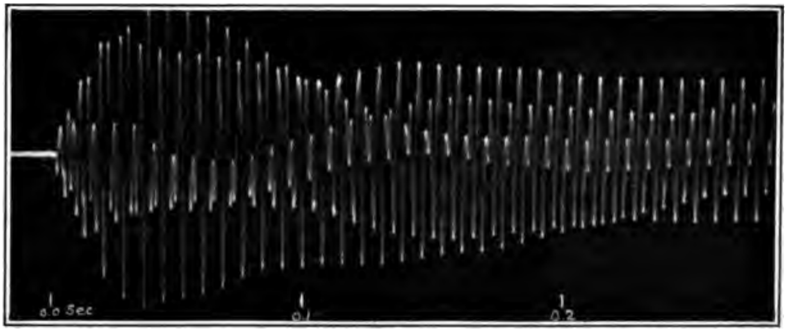


Fig. 158. Photograph of the tone of a piano.

loudest component, but after a tenth of a second, the octave is the loudest part.

The second photograph is of a tone two octaves lower and is of a much more complex nature. There are more than ten partials of appreciable loudness, which are continually changing in relative intensity, due, no doubt, to peculiarities of piano construction which prolong certain partials and absorb others. Whatever complex tone may be generated by the hammer blow, the quality of tone that enters into combination with that from other strings is dependent upon the parts of the tones from the several strings being simultaneously coexistent. The quality of tone obtained from a piano when a melody note is struck is dependent upon the mass of other tones then existing from other keys previously struck and sustained, and it depends upon the length of time each of these tones has been sounding. It is evident that not only does a piano give great variety of tone by various degrees of hammer blow, but there is possible an almost infinite variety of tone quality in combinations of notes struck at intervals of a few hundredths of a second. It is believed that the artistic touch consists in slight variations in the time of striking the different keys, as well as in the strength of the blow, and that tone quality is determined by purely physical and mechanical considerations.

The correctness of this argument is further supported by the mechanical piano players, which attempt to reproduce the characteristics of individual pianists. The more highly developed such instruments become, the more nearly they imitate hand playing in musical effects; in many instances the imitation is practically perfect, and I believe that in the near future the automatic piano will reproduce all of the effects of hand playing.

This condition will in no way displace the artist, nor will it in the least reduce his prestige; on the contrary, it will enhance his standing, and we shall honor him the more for his accomplishments. The machine can never create a musical interpretation, the artist must ever do this.

Sextette and Orchestra

An illustration of very complex tone quality is obtained with the talking machine reproducing the Sextette from "Lucia di Lammermoor," by six famous voices with orchestral accompaniment; photographs of small portions of this music are shown in the frontispiece. The dots on the lower edge of the picture are time signals which are $\frac{1}{100}$ second apart; each line of the picture represents the vibrations due to music of less than one second's duration. On the scale of the original photograph, which is five inches wide, the length of film required to record the entire selection would be 1000 feet. The effects impressed upon the wave by a particular voice or instrument are clearly reproduced; in the middle of the top line, the increase in the amplitude of the wave is due to the entrance of the tenor voice; the second line shows the comparatively simple wave of the solo soprano voice singing high B♭, the smoothness of the curve attesting the pure quality of the voice.

The Ideal Musical Tone

Neither science nor art furnishes criteria which will define the ideal musical tone; a scientific investigation and analysis of the sound from a violin or a piano cannot determine whether it is the ideal. Musical instruments are used for artistic purposes and their selection is ultimately determined by the æsthetic taste of the artist. When an instrument has been artistically approved, the physicist can describe its tonal characteristics and select other instruments possessing the same qualities; he can detect deficiencies and defects and, perhaps, can suggest remedies. "The chemist can scrape the paint from a canvas and analyze it, but he cannot thereby select a masterpiece."

A musical tone of remarkable quality may be produced by a special set of ten tuning forks shown in Fig. 159; these forks are accurately tuned to the pitches of a fundamental tone of 128 vibrations per second and its nine harmonic overtones. When the fundamental alone is sounding, a

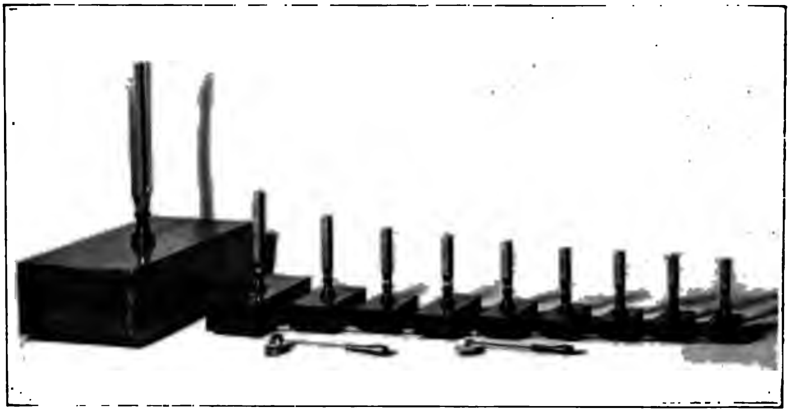


FIG. 159. Set of tuning forks for demonstrating the quality of composite tones.

sweet but dull tone is heard. As the successive overtones are added, the tone grows in richness, until the ten forks are sounding, when the effect is that of one splendid musical tone. One is hardly conscious that the sound is from ten separate sources, the components blend so perfectly into one sound. The tone is vigorous and "living" and has a fullness and richness rarely heard in musical instruments.

Bearing in mind the qualifications just mentioned, one may speak of an ideal musical tone, meaning the most gratifying single tone which can be produced from one instrument. Following the above experiment the ideal tone may be arbitrarily described as one having a strong fundamental containing perhaps 50 per cent of the total intensity, accompanied by a complete series of twenty or more overtones of successively diminishing intensity.

If, while the forks in the above experiment are sounding, they are silenced in succession from the highest downward, the tone becomes less and less rich, until finally the fundamental alone is heard. This is a simple tone and is of a dull, droning quality; the experiment demonstrates that a *pure* tone is a *poor* tone.

It is by no means desirable that all musical instruments should have the quality of tone described. The great variety of musical tone coloring obtained by the modern composers requires instruments of the greatest possible divergence in quality; the contrasts thus available are very effective. Of the instruments of the orchestra, perhaps the horn, in certain of its lower tones, approaches most nearly to the arbitrary ideal.

Demonstration

In the oral lectures, the characteristics of various instruments as described in the preceding pages and as shown by the photographs, were demonstrated by playing the instruments themselves before the phonodeik, which projected the sound waves upon the screen as explained in Lecture III. The sounds so demonstrated were the simple and complex tones from tuning forks, the flute tone as it develops from the simple *pianissimo* quality to the more complex *fortissimo* by the addition of successive overtones, the full and vibrant cornet tone having many partials, the string tone of the violin, with the reversal of phases by changing the direction of bowing, the reedy tone of the clarinet, the varying qualities of vocal tones, the clanging tone from a bell with its interfering inharmonic overtones, and finally the vocal sextette with orchestra, and the concert band, as reproduced by various types of phonographs.

As seen upon the screen, the waves of light, which may be ten feet wide and forty feet long, stretching across the end of the room, are constantly in motion, and pass from one wave form to another, from simple to most complex shapes, with every change in frequency, loudness, or quality of the sound; the wonderfully changing waves flow with perfect smoothness and reproduce visually the harmoniously blending movements of the air, which the ear interprets as music. This ability to see the effects of qualitative changes as well as to hear them is certainly advantageous in an analytical study of sounds, and possibly it adds to the musical effectiveness; it is at least a fascinating and instructive demonstration.

LECTURE VII

PHYSICAL CHARACTERISTICS OF THE VOWELS

THE VOWELS

THE vowels have been more extensively investigated than any other subject connected with speech; the philologist, the physiologist, the physicist, and the vocalist, has each attacked the problem of vowel characteristics from his own separate point of view. The methods of the several classes of investigators, and the expressions of the results, are so unlike and so highly specialized, that one person is seldom able to appreciate them all.

The physicist wishes to interpret the vowels as they exist in the sound waves in air, that is, he wishes to know the nature of the musical tone quality which gives individuality to the several vowels. The tone quality of vowels has been more closely studied than that of all other sounds combined, and yet no single opinion of the cause of vowel quality has prevailed.

The first attempt at an explanation of vowel quality was made in 1829 by *Willis*, who concluded from experiments with reed organ pipes that it depends upon a fixed characteristic pitch; this theory was extended by *Wheatstone* (1837) and by *Grassmann* (1854). *Donders* (1864) discovered that the cavity of the mouth is tuned to different pitches for different vowels. *Helmholtz* (1862–1877) expounded the theory, a development of those given before, that each vowel is characterized, not by a single fixed pitch, but by a fixed region of resonance, which is independent of the fundamental tone of the vowel; this is the so-called *fixed-pitch theory*.

In opposition to this theory, many writers on the subject have held that the quality of a vowel, as well as that of a musical instrument, is characterized by a particular series of overtones accompanying a given fundamental, the pitches of the overtones varying with that of the fundamental, so that the ratios remain constant; this is the *relative-pitch theory*.

Auerbach in 1876 developed an intermediate theory, concluding that both characteristics are concerned, and that the pitch of the most strongly reinforced partial alone is not sufficient to determine the vowel. *Hermann* (1889) has suggested that the vowels might be characterized by partial tones, the pitches of which are within certain limits, but which are inharmonic, the partials being independent of the fundamental. *Lloyd* (1890) considers that the identity of a vowel depends not upon the absolute pitch of one or more resonances, but upon the relative pitches of two or more.

Several quotations will indicate the uncertainty existing at the present time in regard to the nature of the vowels. *Ellis*, the translator of Helmholtz, writes (1885): "The extreme divergence of results obtained by investigators shows the inherent difficulties of the determination." *Lord Rayleigh* (1896) says: "A general comparison of his results with those obtained by other methods has been given by Hermann, from which it will be seen that much remains to be done before the perplexities involving the subject can be removed." *Auerbach* (1909) discusses the various theories, but without deciding which is correct.[83]

PHYSICAL CHARACTERISTICS OF THE VOWELS

Two recent publications on this subject arrive at opposite conclusions. Professor *Bevier* of Rutgers College in one of the most complete studies yet made (1900–1905),[84] using the phonograph as an instrument of analysis, arrives at conclusions in accord with Helmholtz's fixed-resonance theory and the method of harmonic analysis. Professor *Scripture* (1906), formerly of Yale University, says: "the overtone theory of the vowels cannot be correct"; and he gives extended arguments in support of this opinion and opposed to harmonic analysis of vowels.[85] The results of the work here described are in entire agreement with Helmholtz's theory, and they are, therefore, out of harmony with Scripture's arguments.[86]

Standard Vowel Tones and Words

Vowels are speech sounds which can be continuously intoned, separated from the combinations and noises by which they are made into words. A dictionary definition of a vowel is: "one of the openest, most resonant, and continuable sounds uttered by the voice in the process of speaking; a sound in which the element of tone is predominant; a tone-sound, as distinguished from a fricative (rustling sound), from a mute (explosive), and so on."

Helmholtz specifies seven vowels, the "Century Dictionary" gives nineteen vowel sounds in its key to pronunciation, while some writers on phonetics tabulate as many as seventy-two vowel sounds. After preliminary study, eight standard vowels contained in the following words were selected for definitive analysis: f*a*ther, r*aw*, n*o*, gl*oo*m, m*a*t, p*e*t, th*ey*, and b*ee*.

The particular vowels specified are according to the pronunciation of the author. It must be remembered that any change in the pronunciation produces a different vowel, though we may understand the word to be the same, and that the quantitative results would vary for the slightest change in intonation or inflection. Since individual pronunciations vary greatly, even within the range of one language, there seems to be no better method of defining a vowel than by specifying several words, in each of which the author gives the vowel the same sound. Others may disagree with some of the pronunciations, but this does not change the fact that these are the sounds studied and defined in the results. A table of such words follows, while a larger list is given on page 257.

f*a*ther, f*a*r, gu*a*rd
r*aw*, f*a*ll, h*au*l
n*o*, r*o*de, g*oa*l
gl*oo*m, m*o*ve, gr*ou*p
m*a*t, *a*dd, c*a*t
p*e*t, f*ea*ther, bl*e*ss
th*ey*, b*ai*t, h*a*te
b*ee*, p*i*que, mach*i*ne

Some of these sounds are common to all languages; the equivalent of f*a*ther is found in German in *vater*, and in French in *pâte;* the equivalent of n*o* in German is in *wohl*, and in French in *côte;* but there seems to be no equivalent in either German or French for r*aw* or m*a*t.

For the sake of simplicity, instead of using single letters in connection with a multiplicity of signs to designate the several vowels, the writer will give the whole word containing the vowel, the latter being indicated by italics; in pronouncing the phrase "a record of the vowel f*a*ther," one may emphasize and prolong the vowel as "a record of the vowel *fah* . . . ther," or, better, one may pronounce only the vowel part of the last word, as "a record of the vowel . . . *ah* . . . ."

Photographing, Analyzing, and Plotting Vowel Curves

The general procedure in the investigation of a vowel is as follows: the speaker begins to pronounce the appropriate word and prolongs the vowel in as natural a manner as

Fig. 160. Photograph of the vowel *a* in *father*, for analysis.

possible; by means of the phonodeik a photographic record is taken of the central portion of the vowel, while the zero line and time signals are recorded simultaneously with the voice curve. The vowel curve is then analyzed into its harmonic components, corrections are applied, percentage intensities for the several partials are computed, and the results are diagramed, as explained in Lecture V.

A photograph of the vowel father intoned by a baritone voice, at the pitch of $F_2 = 182$, is shown in Fig. 160. The analysis of this curve is given in Fig. 129, page 169, while analyses of other photographs of the same vowel are shown in Figs. 161, 162, and 163.

The ordinates on a vowel diagram indicate the distribution of the energy of the sound with reference to its own harmonic partials. A single analysis gives little information as to the distribution of energy for sounds of intermediate pitches, since the sound analyzed can have no intensity whatever for pitches other than those of its own partials. If the vowel is intoned by the same person at a different pitch, its partials may lie between those of the

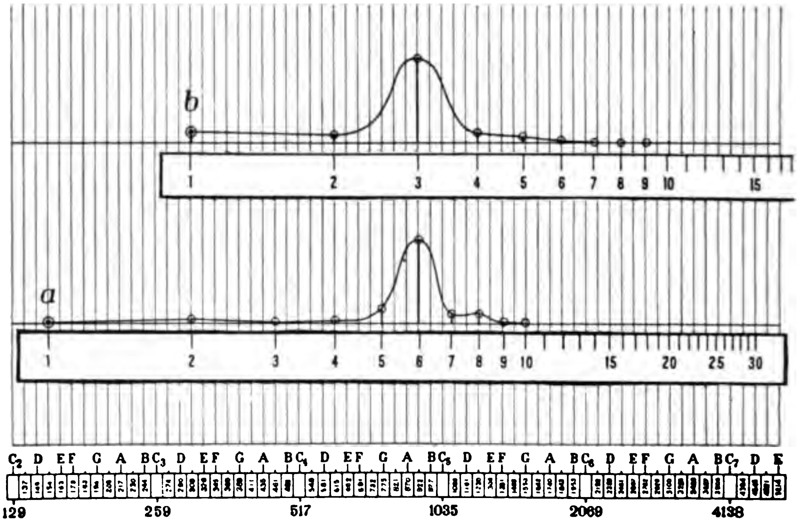


FIG. 161. Loudness of the several components of the vowel *father*, intoned at two different pitches.

first sound as shown in Fig. 161. By plotting many analyses to one base line, Fig. 162, *D*, a curve can be drawn which shows the resonance of the vocal cavities for the particular vowel. For purposes of analytical study it is permissible to show the relations of the separate points of a single analysis to the indicated resonance curve as is done in *A*, *B*, and *C*, Fig. 162. The significance of these curves is more fully explained in the section on Classification of the Vowels, on page 228.

VOWELS OF VARIOUS VOICES AND PITCHES

Each of the eight vowels has been photographed at several pitches as intoned by each of eight voices, giving about a thousand curves, all of which have been analyzed and

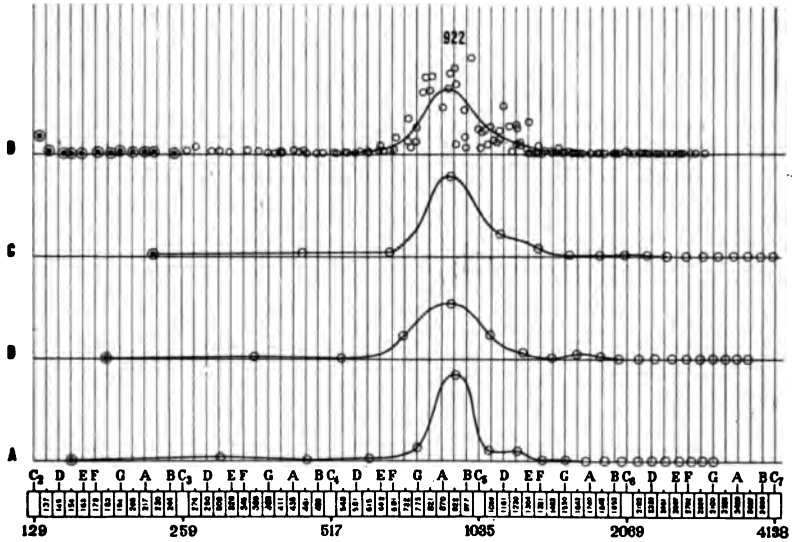


FIG. 162. Distribution of energy among the several partials of the vowel *father*, intoned at various pitches.

plotted. There were two bass voices, two baritones, one tenor, one contralto, one boy soprano, and one girl soprano; the normal pitches of these voices ranged from 106 to 281.

The vowel *father* was intoned by the voice D C M at the pitch $D_2\sharp$ = 155, its energy distribution curve being as shown in the lower part of Fig. 162; the next two curves, *B* and *C*, show the same vowel by the same voice intoned at pitches of $F_2\sharp$ = 182, and $A_2\sharp$ = 227. When the vowel is intoned at the lowest pitch, the sixth partial having a frequency of 930 contains 69 per cent of the total energy of the sound; in the second case the fifth partial of pitch 910 is loudest with 48 per cent of the energy; while in the third case the fourth partial of pitch 908 contains 65 per cent of the energy.

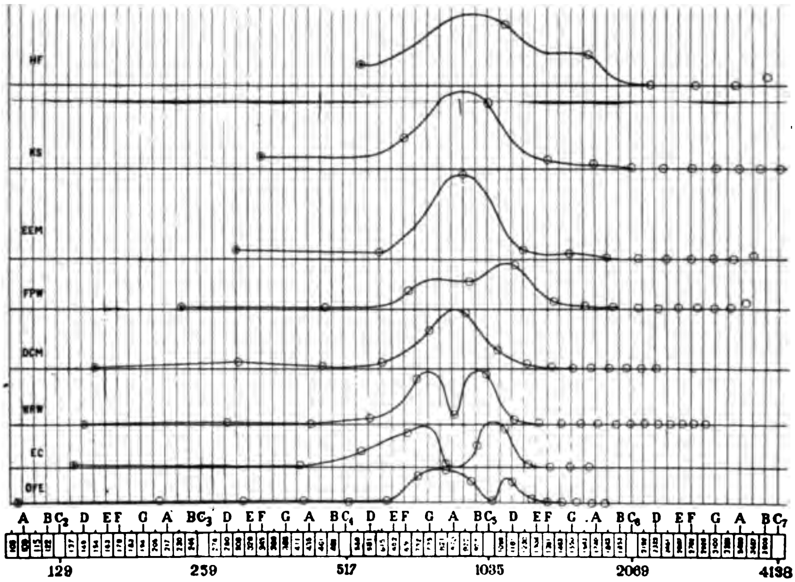


FIG. 163. Distribution of energy among the several partials of the vowel *father*, as intoned by eight different voices.

The same vowel, *father*, was intoned by the same voice, D C M, approximately upon each semitone of the octave from $C_2 = 129$ to $C_3 = 259$, at twelve different pitches; the upper part of the figure, *D*, shows the location of all the component intensities of the twelve analyses; instead of twelve separate curves, one is drawn showing the average energy distribution.

The energy curves of the same vowel, *father*, intoned by eight different voices, at pitches ranging from 106 to 522, are given in Fig. 163. The voices are a bass (O F E), a bass (E C), a baritone (W R W), a baritone (D C M), a tenor (F P W), a contralto (E E M), a boy soprano (K S, 14 years old), and a girl soprano (H F, 10 years old).

The energy curves for the vowel b*ee*, intoned by the same eight voices, at pitches ranging from 111 to 400, are shown

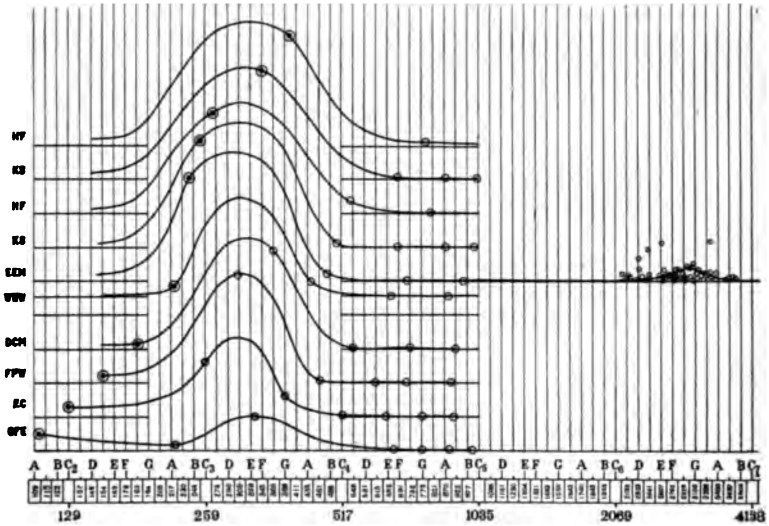


Fig. 164. Distribution of energy among the several partials of the vowel b*ee*, as intoned by eight different voices.

in Fig. 164. For this vowel there are two regions of resonance, one at a pitch of about 300, and the other at a pitch of about 3000. While the greater part of the energy is in the lower resonance, yet it may be said that the higher resonance is the characteristic one, since its absence converts the vowel b*ee* into gl*oo*m, as described on page 231.

These diagrams indicate that there is not a fixed partial which characterizes the vowel, neither is there a single, fixed pitch. The greater part of the energy of the voice is in those partials which fall within certain limits, no matter at what pitch the vowel is uttered, nor by what quality of voice; that is, the vowel is characterized by a fixed region, or regions, of resonance or reinforcement. To establish this theory it is necessary to show that all the different vowels have distinctly different characteristic regions of resonance, which remain the same for all voices. An investigation has been made leading to this conclusion, a detailed account of which is to be published elsewhere; the nature of this study is indicated by the following description, which refers to one voice only.

Definitive Investigation of one Voice

The study of the vowels of different voices and pitches showed that it is practically impossible to obtain the same vowel from the various voices with sufficient certainty to permit of a definitive study, and even extreme variations in pitch for one voice probably alter the accuracy of pronunciation. Therefore it was decided to make a final study of the principal vowel tones of the English language as spoken by one person in order to determine the physical cause of their differences.

Each of the eight vowels previously mentioned was intoned by one voice, D C M; six photographs were made in succession of the vowel spoken in normal pitch and inflection; then six more photographs for the same vowel were made, but approximately on six equidistant tones covering one octave, beginning a little below normal pitch. Thus twelve curves were obtained for each vowel, and for some a larger number was made. There are 202 photographs in this second series, all being made under exactly the same conditions of the speaker's voice and recording apparatus. The curves were analyzed and reduced in one group, a separate energy distribution curve was drawn for each analysis, and finally a composite or average curve was made for each vowel.

Classification of Vowels

The eight final composite vowel curves, drawn on separate pieces of paper, were arranged upon a table and their peculiarities studied. Many schemes of classification were tried, with the final conclusion that all vowels may be divided into two classes, the first having a single, simple characteristic region of resonance, while for the second there are two characteristic regions.

The vowels of the first class are represented by f*a*ther, r*aw*, n*o*, and gl*oo*m. The investigations indicate that the most natural vowel sound and the most elemental words used in speech are m*a* and p*a*, and one of these may be selected as a starting point for a classification. It adds to the effectiveness if the vowels are indicated by simple syllables of the same general form; for the vowels of the first class the words may be m*a*, m*aw*, m*ow*, and m*oo*, or p*a*, p*aw*, p*oe*, and p*oo*h.

The vowels of the second class are represented by m*a*t, p*e*t, th*ey*, and b*ee*; and the new syllables selected are m*a*t, m*e*t, m*a*te, and m*ee*t, or p*a*t, p*e*t, p*a*te, and p*ea*t. This series may be presumed to start from the fundamental vowel m*a*, which for similarity may be expressed by the words m*o*t or p*o*t.

The characteristic curves for vowels of the first class are shown in Fig. 165; the vowels are m*a*, m*aw*, m*ow*, and m*oo*, having maximum resonances at pitches of 910, 732, 461, and 362, respectively. The resonance regions overlap but little; the partials lying within a characteristic region of resonance often contain as much as 90 per cent of the total energy of the sound; there is a conspicuous total absence of higher tones and all the lower tones are weak; the fundamental is of small intensity, containing only about 4 per cent of the energy, unless its pitch lies within a region of characteristic resonance. The vowel *ma* seems to have

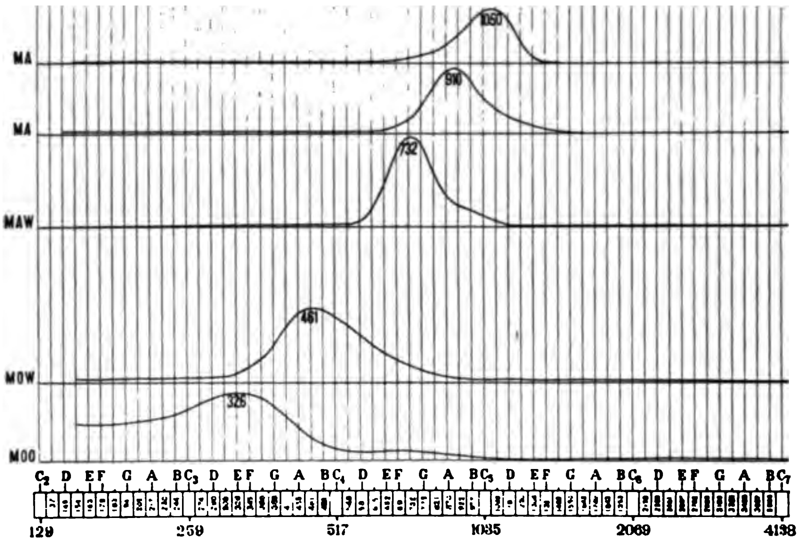


FIG. 165. Characteristic curves for the distribution of the energy in vowels of Class I, having a single region of resonance.

considerable range, as its characteristic may vary from 900 to 1100; two curves are shown for this vowel, as the highest *ma* is perhaps the initial sound from which all vowels are derived. Sometimes this vowel has two resonances close together, as shown in the curve of the second class. The double peak for this vowel is peculiar to certain voices, and probably there is only one resonance, which is separated into two parts by the absence of a particular partial tone from the sound of a particular voice; this condition is indicated by the lower curves in Fig. 163.

The characteristic curves for the vowels of the second class are shown in Fig. 166, each having two characteristic regions of resonance. The curve for m*o*t (m*a*), having a single resonance at the pitch 1050, is placed at the top, and next is a curve for the same vowel in which there are

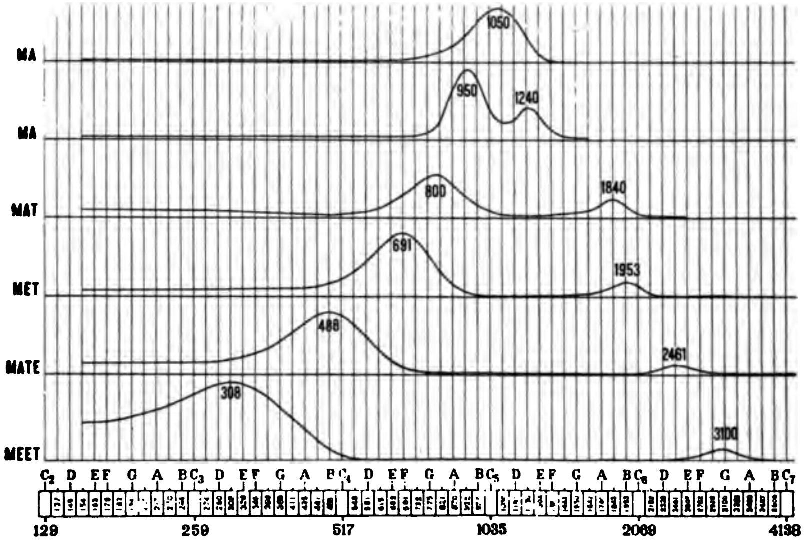


FIG. 166. Characteristic curves for the distribution of the energy in vowels of Class II, having two regions of resonance.

two resonances very close together at pitches of 950 and 1240; the other vowels with the pitches of their resonance regions are: m*a*t, 800 and 1840; m*e*t, 691 and 1953; m*a*te, 488 and 2461; and m*ee*t, 308 and 3100. The lower resonances are practically the same as for the vowels of the first class, but contain only about 50 per cent of the energy, while about 25 per cent is in the higher region. The lower and intermediate tones are stronger than in the vowels of the first class, the fundamental often containing 10 per cent of the energy. Although each vowel is characterized by two regions of resonance, the distinguishing characteristic is the higher resonance.

The characteristic curves show the resonating properties of the vocal cavities when set for the production of the specified vowels, and they have true significance throughout their lengths. These curves may be considered *curves of probability*, or perhaps they may be called *curves of possibility*, of energy emission when a given vowel is intoned. The mouth is capable of selective tone-emission only, that is, the only frequencies of vibration which can be emitted at one time are in the harmonic ratios. If the harmonic scale (see page 169) is placed upon the characteristic curve of a given vowel with its first line at any designated pitch, then the ordinates of the curve at the several harmonic points show the probable intensities of the various partials when the particular vowel is intoned by any voice at the given pitch. These curves show the probable intensity for that part of the energy which is characteristic of the vowel in general, but, since they are averages of many analyses, they do not show the peculiarities of individual voices aside from the vowel characteristic.

Since the pitch region of the maximum emission of energy for a certain vowel is fixed and is independent of the pitch of the fundamental, it follows that the different vowels cannot be represented by characteristic wave forms. When the vowel f*a*ther is intoned upon the fundamental $E_2\flat =$ 154, the sixth partial, $6 \times 154 = 924$, is the loudest, and the wave form has six kinks per wave length. Fig. 167, *a*, is an actual photograph of the vowel f*a*ther from a baritone voice. When the same vowel is intoned by a soprano voice at the pitch $B_3\flat$ = 462, the second partial, 2 × 462 = 924, is the loudest, and the wave shows two kinks per wave length, as in *b*. These curves are for the same vowel, but are wholly unlike. When the vowel *no* is intoned by the baritone at the pitch $B_2\flat$ = 231, the characteristic is the

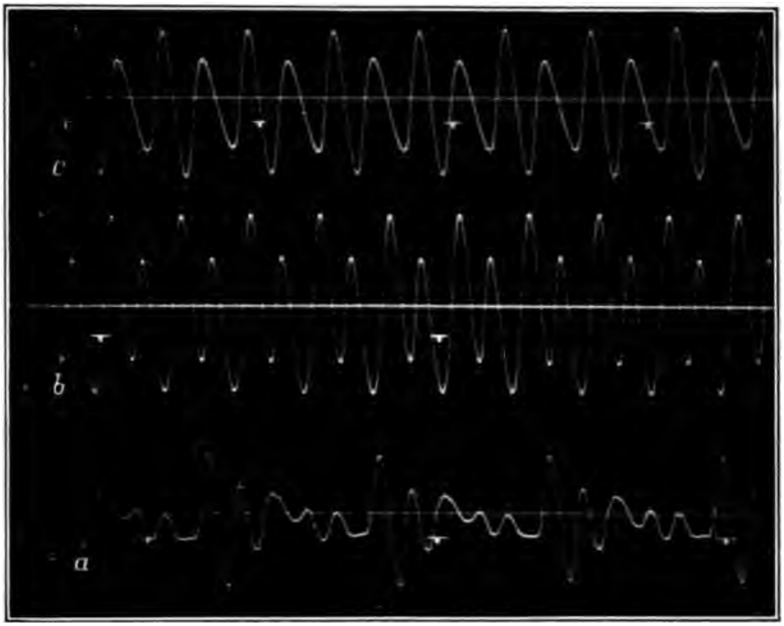


FIG. 167. Photographs *a* and *b*, though unlike, are from the same vowel; *b* and *c* are nearly alike but are from different vowels.

second partial, 2 × 231 = 462; the wave has two kinks and has the appearance shown at *c*. Wave forms *b* and *c* are alike in general appearance, but are for different vowels. The wave form therefore depends upon the pitch of intonation as well as upon the vowel, and one cannot in general determine from inspection alone to what vowel a given curve corresponds. Familiarity with the curves from an individual voice will, however, often enable one to tell what vowel of this voice is represented.

Either one of the word pyramids of Fig. 168 forms an outline for the classification of all vowels; starting at the top and descending to the left are the vowels characterized by single resonances of successively lower pitches; towards the right are those characterized by two resonances, the first of which descends, while the second ascends for the successive vowels. There is a continuous transition from one vowel to the next through the entire range of each class. The number of possible vowels is indefinitely great, having shades of tone quality which blend one into another. It

		p*a* p*o*t						m*a* m*o*t			
	p*aw*		p*a*t				m*aw*		m*a*t		
				p*e*t						m*e*t	
p*oe*					p*a*te	m*ow*					m*a*te
p*oo*h					p*ea*t	m*oo*					m*ee*t

Fig. 168. Word pyramids for classification of the vowels.

is believed that any other vowel from any language after analysis can be placed upon this classification frame as intermediate between some two of those in the pyramid. It happens that the pronunciations used in this study correspond to nearly uniform distribution of resonances, and the vowels are distinct in sound one from another; they form what may therefore be considered a rational selection of standard vowels and give a scientific pronunciation as a basis for word formation and for phonetic spelling and writing.

The continuity of vowel tone, as here described, can be easily demonstrated by intoning the vowel at the top of the pyramid, when, without interrupting the tone, by gradually closing the lips one may cause the intoned sound to pass through all possible vowels of the first class, m*a* . . . m*aw* . . . m*ow* . . . m*oo* (pronounce only the vowels, as, ä . . . a̤ . . . ō . . . o͞o). And again by starting with the same vowel at the top of the pyramid, keeping the lips in constant position, but changing the position of the tongue, one can continuously intone all the vowels characterized by two resonances, m*a* . . . m*a*t . . . m*e*t . . . m*a*te . . . m*ee*t (ä . . . ă . . . ĕ . . . ā . . . ē).

Many photographs have been made which confirm the pyramid classification by simple inspection, showing that

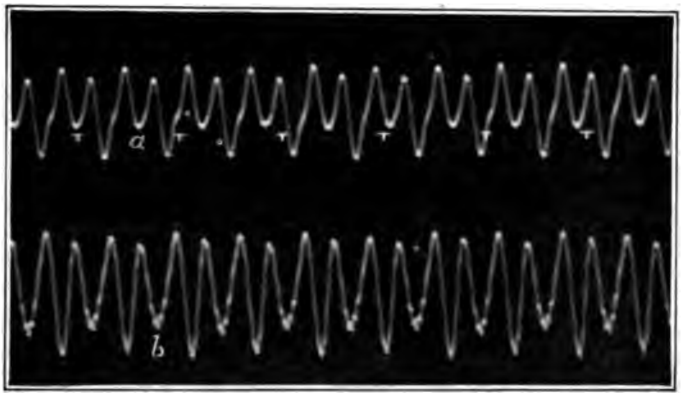


FIG. 169. Photographs of the vowels m*oo* and m*ee*t.

the relations are based upon essential features. Comparative curves for the vowels m*oo* and m*ee*t are given in Fig. 169. The first is for m*oo*, and is a very simple curve, the vowel characteristic being the single resonance which is an octave higher than the fundamental and is represented by the wavelets *a*. The vowel m*ee*t has two characteristic resonances, the first of which, *b*, is practically identical with that for m*oo*, while the second is of very high pitch and is represented by the small kinks which are present throughout the curve but show most clearly near the points *b*; the addition of this high pitch to the sound for m*oo* changes it to m*ee*t.

The relation of the vowels m*oo* and m*ee*t is illustrated by a common difficulty in telephone conversation. Telephone lines are purposely so constructed as to damp out vibrations of high frequency; if the vowel m*ee*t is spoken into the transmitter, its high-frequency characteristic is not carried over the wire, and the sound being heard with this part eliminated is interpreted as m*oo*; for this reason the word *three* is often misunderstood as *two*, to prevent which the *r* in *three* is trilled.

Translation of Vowels with the Phonograph

The phonograph permits a simple verification of the characteristics of certain vowels, since the pitch of the sounds given out by the machine can be varied by changing the speed of the motor which turns the record. When the vowel m*a* is recorded, the greater part of the energy is emitted in tones having a frequency of about 925; if the record is reproduced at the same speed as that at which it was recorded, one hears the vowel m*a*; but when the speed of rotation is reduced so that sounds which previously had the pitch 925 now have the pitch 735, the phonograph speaks the vowel m*aw*; and still further reduction of speed gives the vowels m*ow* and m*oo*.

If m*aw* is recorded, then the record can be made to reproduce m*a* by increased speed of the motor, and the other vowels m*ow* and m*oo* are obtained by a decreased speed as before.

PHYSICAL CHARACTERISTICS OF THE VOWELS

In early experiments with the phonograph the vowels m*a* and m*aw* were recorded several times at various speeds of the cylinder, and afterwards it was impossible to identify the records, because each could be made to reproduce both vowels perfectly.

The vowel m*a* was recorded by the voice D C M on the phonograph, and without stopping the cylinder, the phonograph was made to speak this record into the phonodeik; the sound was photographed and the speed of the phonograph cylinder was determined at the same time with a stop-watch. Analysis of the photograph showed that the fundamental pitch of intonation was 154, while the maximum energy of the sound was in the sixth partial tone, having a frequency of 924; the corresponding speed of the cylinder of the phonograph was one turn in 0.276 second.

The speed of the cylinder was reduced, till the ear judged that the vowel m*aw* was being given by the phonograph; the time for one turn of the cylinder was found to be 0.348 second, corresponding to a frequency of 730 for the tone of maximum energy. Similar trials were made for the vowels m*ow* and m*oo*, and the results of all the experiments are shown in tabular form.

Vowel	m*a*	m*aw*	m*ow*	m*oo*
Speed, sec. per turn	0.276	0.348	0.572	0.814
Phonograph, *n*	924	730	444	311
Analysis, *n*	922	732	461	326

Frequencies of vowels obtained by translation with the phonograph.

The characteristics from the phonograph experiments for m*a*, m*aw*, m*ow*, and m*oo*, are 924 (the original record), 730, 444, and 311, respectively, while the photographic analyses give 922, 732, 461, and 326.

Since in this experiment the pitch of the fundamental is lowered in the same proportion as is that of the characteristic, to the abnormally low pitch of 54 for m*oo*, it is better to record the first tone at a pitch higher than that of normal speech, or to make the record from a contralto or soprano voice.

If the vowel m*a* is recorded at different pitches or by different voices, when the speed is changed so that any one pitch or voice gives m*ow*, all other pitches and voices give m*ow* at the same time, since the characteristic for each vowel is the same (approximately) for all voices and pitches. A similar relation exists when records of any one of the vowels m*a*, m*aw*, m*ow*, and m*oo* are translated by changed speed of reproduction to any other one of these vowels.

In order that the loudness of a sound may remain constant when the pitch is lowered, the amplitude should increase, as was explained in Lectures II and V, in the proportion shown in Fig. 113. In phonographic reproduction, the amplitudes of the several component tones remain constant as the frequency is reduced, since the amplitudes are determined by the depth of the cutting in the wax; this causes a diminution in intensity proportional to the square of the speed reduction, and alters the *relative* loudness of the several component tones; hence a translated vowel often has an unnatural sound, though it retains the vowel characteristic. This difficulty is somewhat overcome by the method of experimentation described below.

When the vowel m*ow* is intoned by a baritone voice at the normal pitch for speech, $E_2\flat = 154$, the characteristic is the third partial of pitch $3 \times 154 = 462$. If m*a* is recorded on the phonograph by the same voice at the same pitch, the characteristic is the sixth partial of pitch $6 \times 154 = 924$; when the phonograph speed is reduced to sound m*ow* from this record, the fundamental pitch becomes 77, and the characteristic is the sixth partial of this lower pitch, $6 \times 77 = 462$; while this sound is clearly m*ow*, it is not like a natural m*ow* of the baritone voice, being pitched on such a sub-bass fundamental. When m*a* is recorded by a contralto voice on the fundamental $E_3\flat = 308$, the characteristic is the third partial of the pitch $3 \times 308 = 924$; if now this record is reproduced at a slower speed to give m*ow*, the fundamental falls to $E_2\flat = 154$, and the characteristic is still the third partial of pitch $3 \times 154 = 462$; this translated m*a* of the contralto voice becomes m*ow* of the *baritone voice* in general quality, and has a natural sound.

Phonographic translation of the vowels of the second class, m*a*t, m*e*t, m*a*te, and m*ee*t, is not possible, for each has two regions of resonance, as is shown in Fig. 166, the higher increasing in frequency when the lower decreases.

Whispered Vowels

The vowels can be distinctly whispered without the production of any larynx tone, that is, without fundamental or pitch and without the series of partials which determine the individuality of the voice, but these whispered sounds must contain at least the essential characteristics of the vowels. Photographs of such whispered vowels are readily obtained and, the time signals being photographed simultaneously, they give by direct measurement the absolute pitch of the vowel characteristics.

Many photographs have been taken of nine vowels, whispered by several voices; inspection of these at once proves the general correctness of the classification of the vowels already given.

Whisper records for the four vowels of the first class m*a*, m*aw*, m*ow*, and m*oo*, are shown in Fig. 170, m*a* being at the top. Since the usual voice tones are entirely absent, each curve consists mainly of one frequency, that charac-

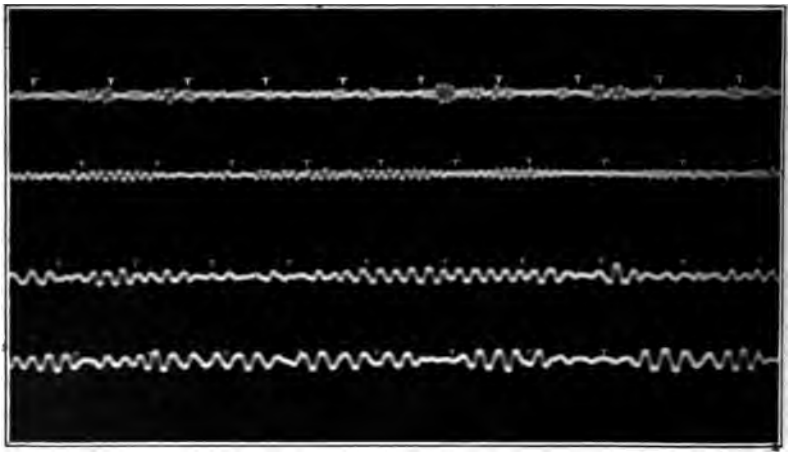


FIG. 170. Photographs of whispered vowels of Class I.

teristic of the vowel. These curves are arranged in the order of the classification and it is evident that the frequency of the principal vibration in each increases from the lower to the upper record.

The second class of whispered vowels, m*a*t, m*e*t, m*a*te, and m*ee*t, is shown in Fig. 171, m*a*t being at the top. Each curve of this group has two distinct frequencies; the curve for m*ee*t has the lowest and the highest, the high frequency being superposed on the lower like beads on a string; the other curves in order show that the lower tone increases in frequency, while the higher one decreases, the two being quite entangled in the upper curve for *mat*.

The variation in the wave form, which is not periodic and therefore is not caused by beats, is perhaps due to the fact that, as the vocal cords are not in motion, the vibration of the air in the mouth cavity is uncontrolled and fluctuates in both intensity and pitch within the characteristic limits.

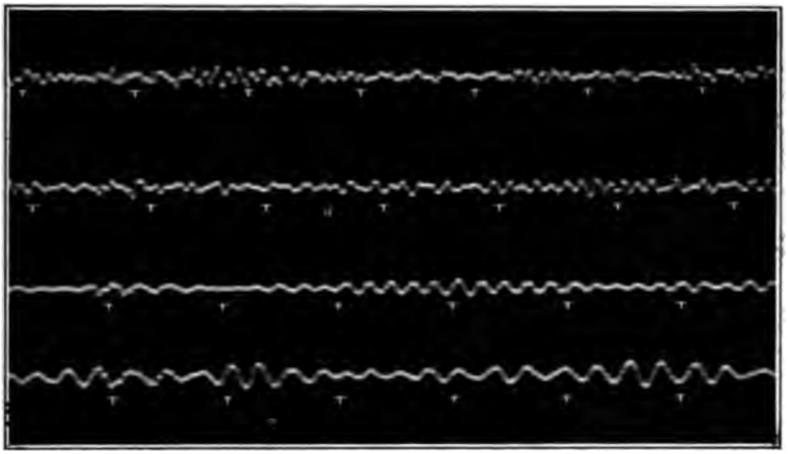


FIG. 171. Photographs of whispered vowels of Class II.

Comparisons of forty-five curves of whispers show frequencies for the characteristics as given in the table; the

VOWEL	*ma*	*maw*	*mow*	*moo*	*mat*	*met*	*mate*	*meet*
Whisper, *n*	1019	781	515	383	857 1890	678 1942	488 2385	391 2915
Analysis, *n*	922	732	461	326	800 1843	691 1953	488 2461	308 3100

Frequencies of the characteristics of whispered vowels.

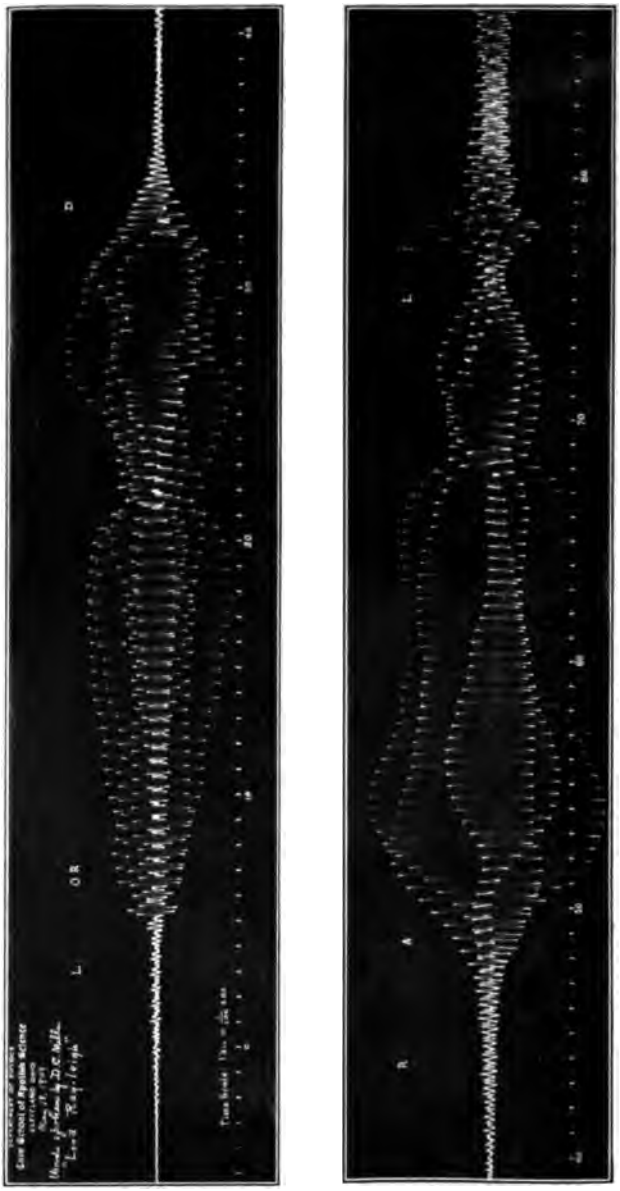


FIG. 172. Photograph of the words "Lord Rayleigh" as spoken by a baritone voice.

frequencies determined by analysis of the spoken vowels are also given for reference. It appears that the resonance frequency of the mouth in whispering is somewhat higher than when speaking, though the whisper characteristics are well within the limits of those of speech.

Theory of Vowel Quality

The analytical studies which have been described lead to the conclusion that intoned vowels are strictly periodic or musical sounds. It is unusual, however, to prolong a vocal sound without variation; in song a sustained tone is usually given some emotional expression, and in spoken words the vowels change continuously and are blended with the consonants. The photograph of the soprano voice shown in the frontispiece represents a sustained musical tone which, though simple in quality, is continually changing in intensity. The flowing of speech tones from one quality to another is illustrated by the photographs, Fig. 172, of the spoken words "Lord Rayleigh," and Fig. 184, of the words "Lowell Institute." The slightest change in the sound causes a change in the wave form. It requires some practice, but it is not impossible to maintain a pure vowel tone unchanged for several seconds, in which time there may be hundreds of waves which are truly periodic; Fig. 173 shows the periodicity of the vowel sound m*a*te. Such a photograph is a complete justification of the application of harmonic analysis to the study of vowel curves.

The mouth with its adjacent vocal cavities is an adjustable resonator; by varying the positions of the jaws, cheeks, tongue, lips, and other parts, this cavity can be tuned to a large range of pitches. When the mouth is wide open and the tongue is low, the cavity responds to a single pitch of high frequency, and is set for the vowel f*a*ther, *A*, Fig. 174; when the opening between the lips is small, $\bar{o}\bar{o}$, the pitch is lowered, as for gl*oo*m. The mouth cavities may be adjusted to reinforce two different pitches at one time, as has been explained by Helmholtz; when set for the vowel m*ee*t,

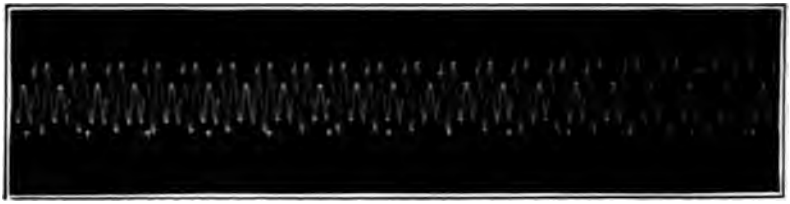


FIG. 173. Photograph of the vowel *mate*, showing periodicity of the wave form.

E, the cavity responds to two simple tones, one corresponding to the back part of the oral cavity, and the other to the channel between the tongue and the roof of the mouth. The resonance of the mouth may be illustrated by holding forks of certain pitches before the mouth, when it is set for the vowel, but when no sound is made; the fork tone

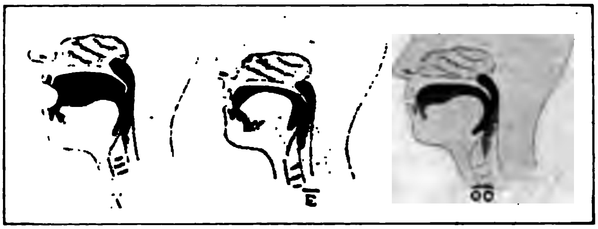


FIG. 174. Shape of mouth cavities when set for various vowels.

will be strongly reinforced. Fig. 175 shows five forks corresponding to certain vowel pitches as determined by Koenig,[87] together with brass resonators tuned to these pitches, and therefore producing the same effect as the mouth when set for the several tones.

The variation in the size of the mouth cavities, as between adults and children, is easily compensated by changing the opening of the lips. The effect of such a resonator, having certain natural periods, can only be to modify the intensity and phase of the several components which are already present in the sound produced by the vocal cords; the resonator cannot originate any tone. If the frequency

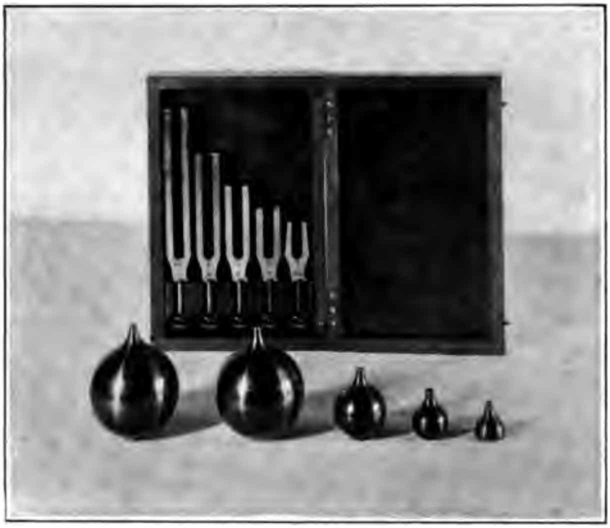


FIG. 175. Koenig's forks and resonators for vowel characteristics.

of the resonating cavity coincides with that of one of the partial tones of the voice, the effect must be to reinforce this particular partial; if the pitch to which the mouth cavity is set does not exactly coincide with any partial of the voice, then those partials whose pitches approximate that of the mouth will still be favored. Both of these conditions are illustrated by the curves of the eight voices for *father*, Fig. 163.

It is necessary that the sound generated by the vocal cords should be a composite containing at least those partials which are characteristic of the vowel to be spoken. The sounds of the voice are normally very rich in partials; in one analysis of the vowel m*a*t was found every partial from one to twenty inclusive; in another analysis of the same vowel, there are eighteen partials, the highest being number twenty-four; in the vowel m*e*t, an analysis shows sixteen partials, the highest being number twenty-three.

Peculiarities of individual voices are probably due to the presence or absence of particular overtones in the larynx sound, according to incidental or accidental conditions. A low voice of a man has a large number of partials not essential to the vowel, which, so to speak, overload the characteristic tones; these partials may make the voice louder, but they detract from clearness of enunciation. A child's voice, on the contrary, produces only the higher tones, and but few besides those necessary for the vowel; the enunciation is, therefore, especially clear, clean-cut, and distinct. One is conscious of the greater clearness of enunciation of a child's voice when listening to a conversation in a foreign language which is understood with difficulty.

The process of singing a vowel is probably as follows. The jaws, tongue, and lips, trained by lifelong practice in speaking and singing, are set in the definite position for the vowel, and the mouth is thus tuned unconsciously to the tones characteristic of that vowel. At the same time the vocal cords of the larynx are brought to the tension giving the desired pitch, automatically if one is trained to sing in tune, but usually as the result of trial. When the air from the lungs now passes through the larynx, a composite tone is generated, consisting of a fundamental of the given pitch accompanied by a long series, perhaps twenty in number, of partials, usually of a low intensity. The particular partials in this series which are most nearly in unison with the vibrations proper to the air in the mouth cavity, are greatly strengthened by resonance, and the resultant effect is the sound which the ear identifies as the specified vowel sung at the designated pitch.

If, while the mouth cavity is maintained unchanged in position, the vocal cords are set successively to different pitches and the voice is produced, then one definite vowel, the same throughout, is recognized as being sung at different pitches. In this case the region of resonance is constant, though the pitch of the fundamental may vary, as may also the pitch and order of the particular partials which fall within the region of resonance.

It follows that a vowel cannot be enunciated at a pitch above that of its characteristic, a condition which is easily shown to be true for those vowels having a low-pitched characteristic, such as gl*oo*m. Words which are sung are often difficult to understand; this may be due in part to the fact that the tones of the singing voice are purer than those of speaking, that is, that they have fewer partials; also, the words must be intoned upon pitches assigned by the composer, and the overtones may not correspond in pitch with the characteristics of the vowel; furthermore, singing tones are often too high to give the characteristic, even approximately, as is explained in the next lecture.

LECTURE VIII

SYNTHETIC VOWELS AND WORDS, RELATIONS OF THE ART AND SCIENCE OF MUSIC

Artificial and Synthetic Vowels

The most convincing proof of a vowel theory would be a reproduction of the several vowels by compounding the partial tones obtained in the analyses. Marage of Paris has obtained vowel sounds by means of artificial larynxes and mouth and nasal cavities, Fig. 176, combined with artificial lungs made of bellows and an electric motor. Such an apparatus, like the doll that says "ma-ma," is very interesting, but it gives no evidence regarding any particular theory of vowel quality; the vowels so made are not synthetic reproductions scientifically constructed, but are more properly imitations.[88]

Fig. 176. Apparatus for imitating the vowels.

Koenig devised the wave siren, a simple form of which is shown in Fig. 177, for reproducing any desired wave motion, the shape of the wave being cut on the edge of a disk; he also made a large wave siren for compounding sixteen sim-

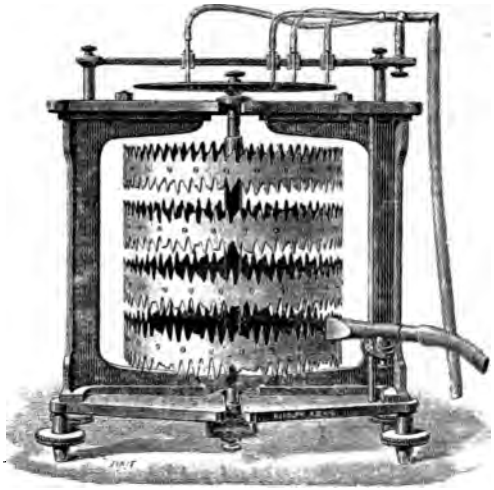


FIG. 177. Koenig's wave siren.

ple tones of variable loudness and phase; Fig. 178 shows this apparatus.[89] These instruments have not proved successful, perhaps because the resulting wave in air is not a reproduction of that cut on the disks.

One of Helmholtz's most celebrated devices is the apparatus for reproducing vowels synthetically by means of tuning forks.[90] It consists of ten electrically driven tuning forks in connection with adjustable resonators, Fig. 179. Helmholtz describes the experiments by which he says he produced a very fine *no*, a good

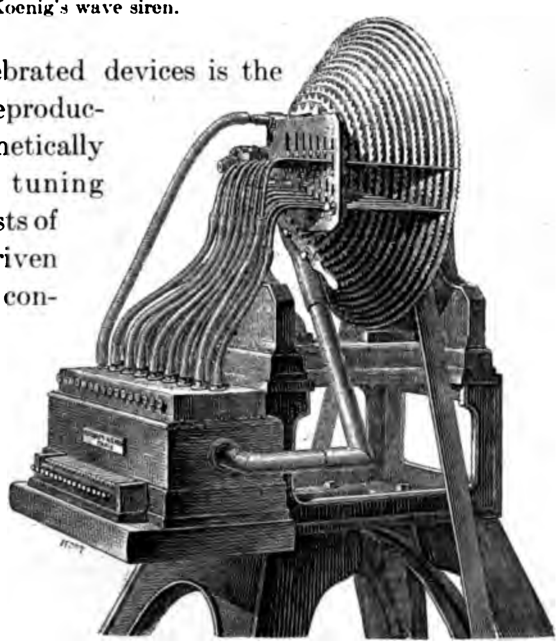


FIG. 178. Koenig's wave siren for compounding sixteen partials.

gl*oo*m, and a passable *raw*, while the other vowels could be only imperfectly imitated. Zahm, who repeated the experiments with the Helmholtz apparatus, remarks that the resemblance of the artificial sounds to the natural ones is, at best, more or less fanciful. Rayleigh says: "These experiments are difficult and do not appear to have been repeated."

Helmholtz explains the difficulties as in part due to the fact that the higher forks give only weak tones and that the series is not large enough; his highest fork gave 1792 vibrations per second. Other difficulties are that Helmholtz and others have not known the exact composition of any of the vowels; and, had the composition been known, there was no adequate method of adjusting the several components to the proper intensity. In order to imitate an actual vowel, it is desirable that the pitch of the fundamental shall correspond exactly to that of the voice being reproduced. Helmholtz had only one series of forks giving eight partials based on 112 vibrations per second, which was later extended to give also eight partials based on 224 vibrations.

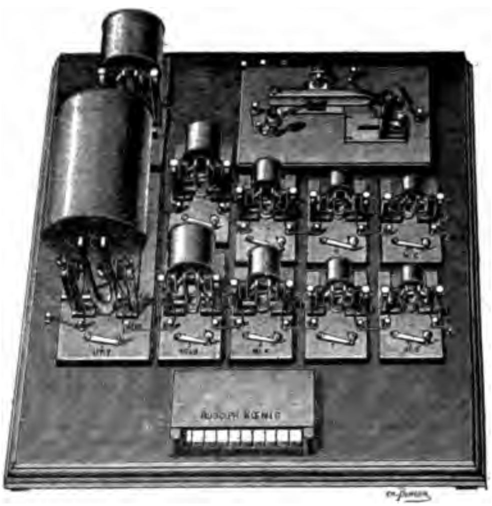


FIG. 179. Helmholtz's tuning-fork apparatus for compounding ten partials.

Helmholtz also tried organ pipes, and says: "We can effect our purpose tolerably well with organ pipes, but we must have at least two sets of these, loud open and soft stopped pipes, because the strength of tone cannot be increased by additional pressure of wind without at the same time changing the pitch."

Our study of organ pipes showed that these difficulties are not insurmountable and that pipes can be made which are more advantageous than tuning forks for experiments in synthesis.

The most suitable pipes are stopped pipes of wood, known as "Tibia" pipes; these are of large cross section in proportion to length, have narrow mouths, and are voiced for low wind pressure. The pipes have lead "toes," the openings in which can be made larger or smaller, thus adjusting the quantity (pressure) of the air entering the pipes. This adjustment in connection with that of the lip and throat permits any strength of tone to be obtained from the least to the full tone. Every change in the strength of tone causes a change in pitch which must be compensated by adjusting the stopper. Many analyses show that the tones from such pipes have 99 per cent of fundamental, that is, practically, the tone is simple.

A vowel having been photographed and analyzed, the synthesis can be performed in a strictly quantitative manner by means of the phonodeik. A set of pipes is prepared, one pipe for each partial of the given vowel; for the vowels as spoken by D C M the least number of pipes is six for the vowel gl*oo*m, while the vowel f*a*ther requires ten pipes, and the vowel m*a*t sixteen; the group of pipes for the latter is shown in Fig. 180.

Even sixteen pipes reproduce only the more important partials of the vowel m*a*t, since the full analysis shows twenty or more component tones in some instances. For voices of higher pitches the number of pipes required is less. Each set of pipes is mounted as compactly as possible on a separate,

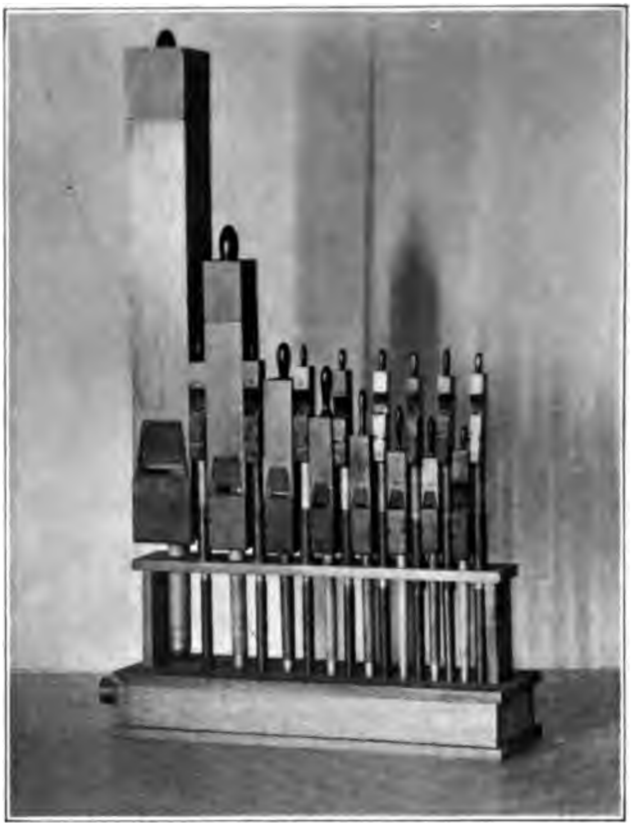


Fig. 180. Group of organ pipes, which, when sounded simultaneously, reproduce the vowel ă in *mat*.

small wind chest. The smaller pipes alternate with the larger ones and their mouths are on a different level to prevent interference.

The analysis of the voice curve gives the actual "observed" amplitudes of the several components of the curve. The set of pipes is now placed in front of the phonodeik in the position occupied by the original voice, and each pipe corresponding to a component tone is separately adjusted, till it shows in the phonodeik the amplitude required by the analysis for this partial; the adjustment is readily verified as the amplitude is directly measurable on the ground glass of the camera. The reproduction is wholly independent of the peculiarities of the phonodeik, for it is made with the same instrument and under the same conditions as was the original record.

The fundamental pitch is set from a piano or tuning fork, and the other partials can then be tuned to the exact harmonic ratios by means of the phonodeik. The pipe for the second partial, already in approximate tune, is sounded simultaneously with the fundamental, the resulting curve is observed on the ground glass of the phonodeik, and the pipe is tuned until the wave form remains constant. The tuning is now necessarily perfect, since inexact relationship produces a slowly changing curve. The third and other partials are then successively tuned in the same manner.

This method of tuning is perhaps the best possible for two or more frequencies which are in exact ratios, since it possesses the advantages of Lissajous's optical method and is more generally applicable. While it is not difficult to adjust a small number of pipes to practically perfect harmonic frequencies, it is hardly possible to tune sixteen pipes so that the resultant wave form remains unchanged, and consequently the synthesized tones do not blend as perfectly as do the partials from a single source. With care, however, the tuning is sufficiently exact to secure success in the experiments.

The set of tones thus obtained, when sounded simultaneously in front of the phonodeik, must give a resultant wave which can be analyzed into the same components as those of the original voice curve; the inference then is that the composite sound will give to the ear the same vowel sound as did the original voice.

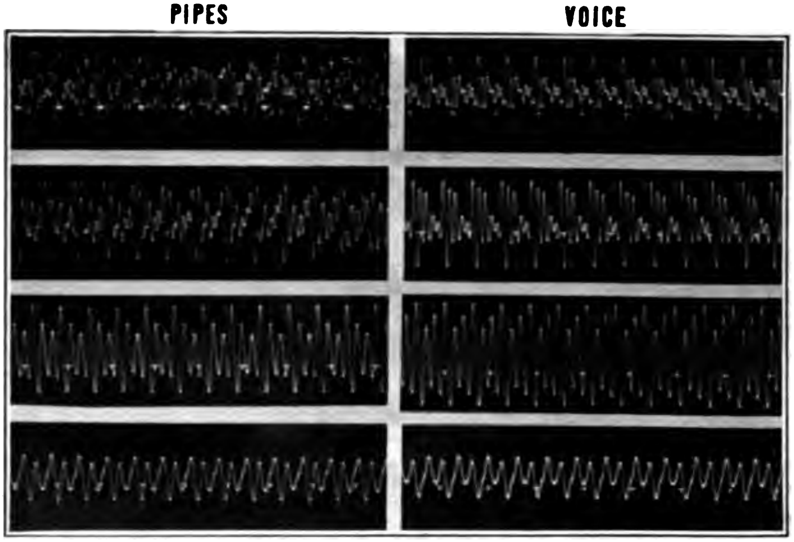


FIG. 181. Photographs of the vowels *ma*, *maw*, *mow*, and *moo*, as intoned by the voice and as synthetically reproduced with organ pipes.

Fig. 181 shows several curves from synthetic pipe-vowels together with corresponding voice curves; there is a close resemblance between the two. In the more complex pipe-vowels there is a continual change in wave form, produced by a slowly shifting phase, due to imperfect tuning.

In the oral lectures demonstrations were given of the synthetic pipe-vowels constructed from the analyses of the speaker's voice, and the pipes and voice were sounded alternately to permit a direct comparison. There is no difficulty in identifying the synthetic vowels and in detecting the peculiar qualities of the voice. In this manner the general characteristics of the vowels f*a*ther, r*aw*, n*o*, and gl*oo*m, spoken by D C M, are reproduced. The synthetic vowels m*a*t and b*ee* were demonstrated, the latter showing the transformation of *o͞o* to *ēē*. The experiments also included the four different syntheses of the vowel f*a*ther, and the reproduction of the simple words *ma-ma* and *pa-pa,* as described in the next section.

In the laboratory eleven vowels have been successfully reproduced by organ-pipe synthesis, the several groups of pipes used being shown in Fig. 182. The same methods which make it possible to construct the vowels synthetically, of course, enable one to reproduce the tone quality of any orchestral instrument; in this manner the tones of the flute, the clarinet, the violin, and the oboe have been imitated.

Word Formation

Experiments with synthetic vowels lead to interesting conjectures concerning the origin of speech. The analyses of vowels show that the highest-pitch, articulate sound of the human voice is *ah* as in f*a*ther; if while intoning this vowel, the lips are closed and opened alternately, the result is the word *ma-ma;* if the nasal passage is also closed the word is *pa-pa*. Perhaps the automatic production of these sounds by the infant when in need of attention explains the origin of these names for the parents. Our organ-pipe talking apparatus, which is, indeed, but a mere infant, can also say these words.

Referring again to the diagram which shows the analyses for the vowel f*a*ther from eight voices, Fig. 163, we will select four, the voices of a child, a boy, a woman, and a man,

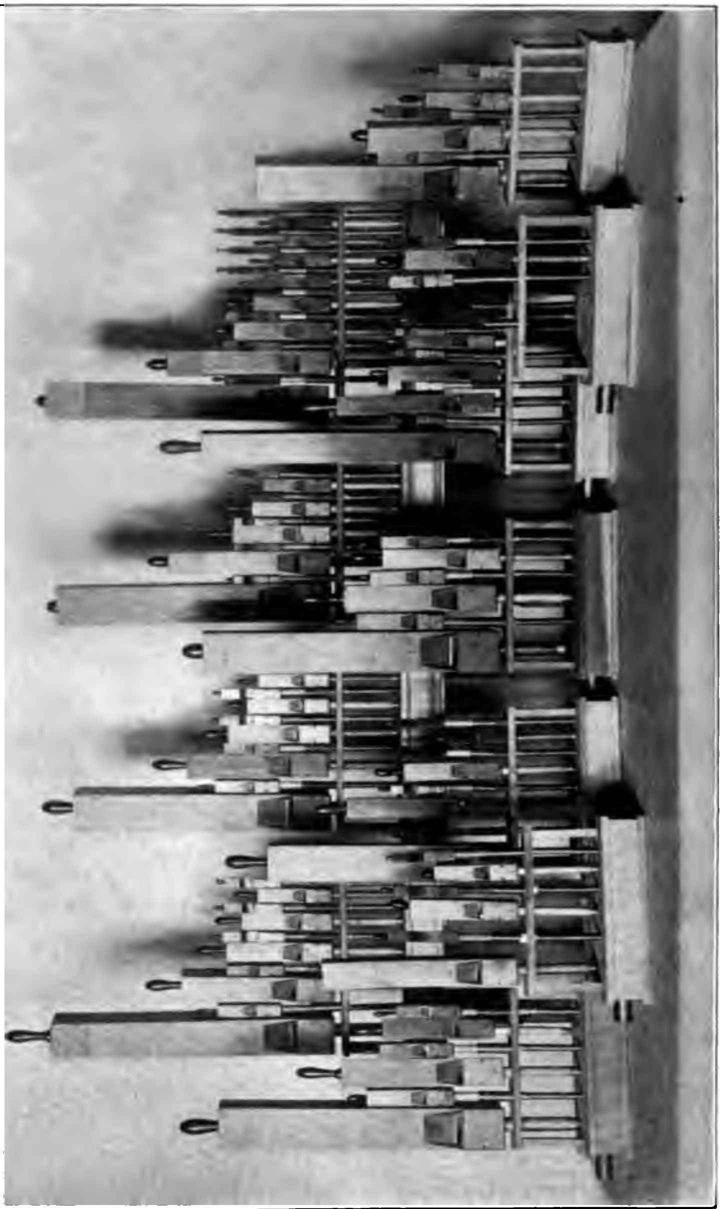


FIG. 182. Sets of organ pipes for the synthetic reproduction of the principal vowels.

for synthetic reproduction. The four sets of pipes shown in Fig. 183 reproduce this vowel, f*a*ther, as intoned by the four different voices. For the child's high-pitched voice only three pipes are required; when these are sounded the vowel f*a*ther is very clearly produced.

If the supply of air from the "lungs" to the "throat" is stopped and released by pressing the edge of the hand on the rubber supply tube, the "infant" is caused to cry *pa-pa*

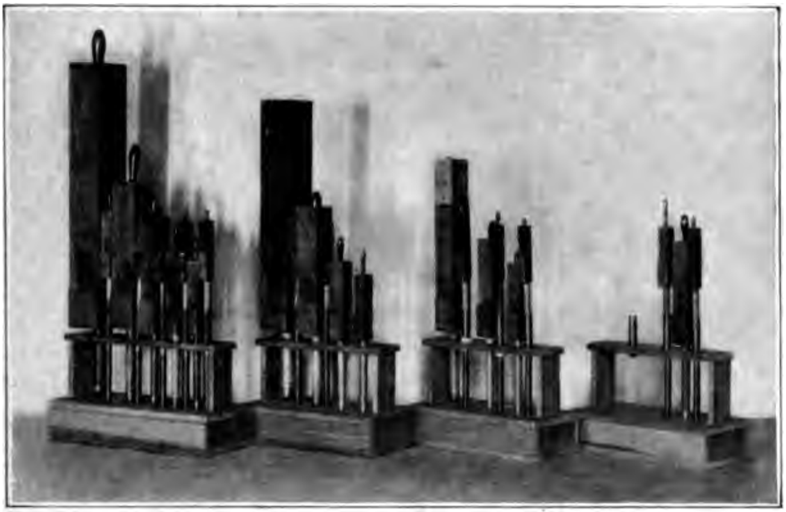


FIG. 183. Sets of pipes, reproducing the vowel *a* in *father*, as intoned by four different voices.

with a voice that would be considered human, if the source were unknown. Similarly the pipes for the contralto voice, six in number, say *pa-pa* perhaps even more naturally, while the boy's voice, eight pipes, and the baritone, ten pipes, speak the same word very clearly; the latter being the speaker's voice, the word was actually spoken for comparison. By thus using the vowels in a word, the success of the synthetic reproduction is the more easily recognized.

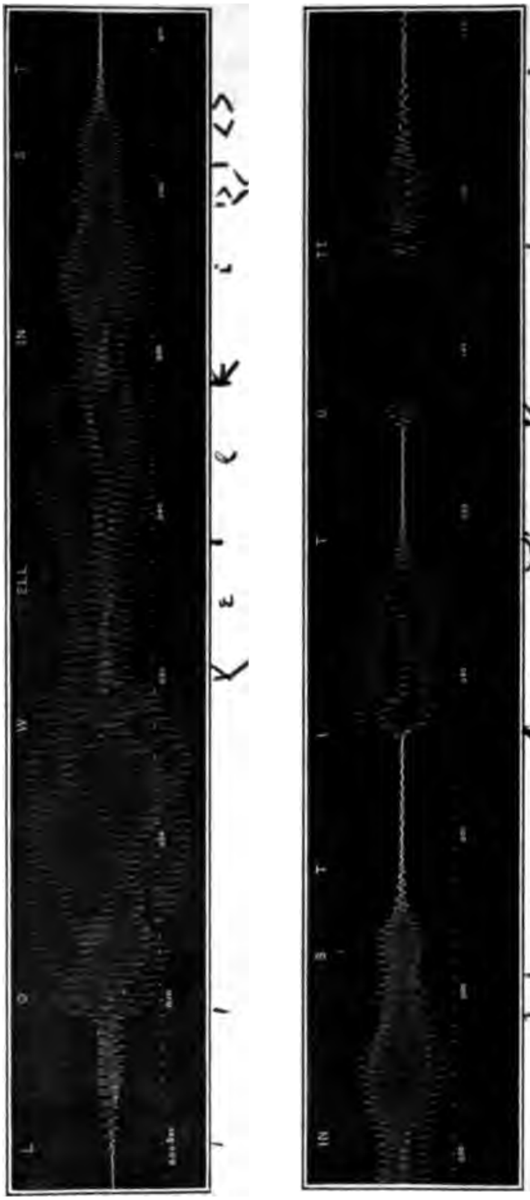


FIG. 184. Photograph of the words "Lowell Institute" spoken by a baritone voice.

While the four sets of pipes are entirely different, containing from three to ten pipes each, and while the general tone qualities are very different, yet each contains the same vowel characteristic as its loudest component, and each distinctly sounds the vowel f*a*ther. The comparison of the four is a striking illustration of the truthfulness of the analyses.

If the flow of air is only partially interrupted, the sound does not altogether cease between syllables, and the pipes pronounce *ma-ma;* this is better shown by alternately producing *pa-pa* and *ma-ma.*

If the same experiment is made upon the synthetic vowel r*aw*, the word *paw-paw* is spoken, and if the vowel m*a*t is used, the word *pă-pă* is obtained; thus we have the three pronunciations, *pa-pa*, *paw-paw*, and *pă-pă.*

It is certainly possible, by further experimentation, to produce various other noises, explosive effects, hisses, etc., which when combined with the vowel tones already produced will form other words, as *sss-ee*, *bb-ee*, *a-tt*, *nn-o*, etc.

In addition to the imperfect tuning the absence of inflection and tone flow gives an unnatural effect to the synthetic words. In speaking the word *pa-pa*, for instance, the two syllables are usually uttered at different pitches, and in any phrase there is an almost continual change of quality and quantity of tone, as is shown in Fig. 184, which is a photograph of the words "Lowell Institute" as spoken by D C M. These words were spoken in a natural manner, in about one and two tenths seconds. The first word and part of the second, represented by the three syllables *low-ell-ins*, consist of a continuous flow of sound, a kind of "melody of vowel tone." In contrast, the word *ins-ti-tute* is disrupted by the *t's* which consist of interruptions of the sound lasting about a tenth of a second each.

The exact form of the record depends not only upon the words spoken, but also quite as much upon the peculiarities of the individual voice and upon the resonance characteristics of the recording apparatus. The incidental variations and the extensive vocabulary of language, would produce an enormously extended set of records, yet it is possible to learn to "read" the sound-wave records of spoken words; such records constitute a system of phonetic writing, but the system is neither "shorthand" nor "simplified spelling."

Such records are, doubtless, capable of being transformed into corresponding movements of the air, and thus the original sound may be reproduced after the manner of the talking machine.

These experiments suggest a scientific method of word formation. A series of eight distinctly different vowels has been established by quantitative analysis; these vowels may be combined with the various consonant, explosive, and aspirant sounds according to a systematic scheme, forming sets of words each having a distinct pronunciation. Any word formula having been arbitrarily selected, a group of eight words may be formed, as shown in the following table. If the sound represented by the letter *m* is placed before each vowel, we have the words in the first column; words thus formed, which have not been in use in the English language, are printed in italics. If to each of these words is added a final *s*, the words of the second column are obtained, while the addition of a *t* gives the words in the third column. If the word formula chosen is that for ***bottle***, the eight words of the sixth column are formed, only three of which are in use. Other combinations are shown in the table.

Such a scheme of word formation contains interesting suggestions for uniform and simplified spelling, and also for a uniform pronunciation.

I	ma	*mas*	mot	baa bah	bot	bottle	fot
II	maw	moss	maught	ba baw	bought	*bawtle*	faught
III	mow	mose	moat mote	beau bow	boat bot	*boatle*	*foat*
IV	moo	moos	moot	booh	boot	*bootle*	*foot?*
V	*ma*	mass mas	mat	*ba*	bat	battle	fat
VI	*me*	mess	met	*be*	bet	*bettle*	fet
VII	may	mace	mate	bay	bate bait	*baitle*	fate
VIII	me	meese mease	meet meat	be bee	beet beat	beetle	feet feat

I	pa	pot pawt	ha	hot	hock	rah	roc rock	tot
II	paw	paut	haw	haught	hawk	raw	*rawk*	taut taught
III	poe	pote	ho hoe	*hote*	hoke	roe row	roak	tote
IV	pooh	*poot*	hoo who	hoot	*hook?*	rue	*rook?*	toot
V	*pa*	pat	*ha*	hat	hack	*ra*	rack wrack	tat
VI	*pe*	pet	*he*	het	heck	*re*	reck wreck	tet
VII	pay	pate	hay	hate	hake	ray re	rake	tate tait
VIII	pea	peat	he	heat	*heek*	ree	reek wreak	teat

Vocal and Instrumental Tones

The difference between vocal and instrumental tones is exhibited by the analyses shown in Fig. 185, which illustrates the *fixed-pitch theory* and the *relative-pitch theory* of tone quality. The upper lines of the figure show the relative loudness of the several partials for the vowel father, intoned at three different pitches; the maximum resonance has a *fixed pitch* of about $A_4\sharp = 922$, the loudest partial being in turn the sixth, the fifth, and the fourth as the pitch of the fundamental rises. The lower lines of the figure show flute tones at three different pitches. The loudest component is always the second partial, which changes pitch with the fundamental; thus it is the *relation of partials* that is constant, and not the pitch of the partial.

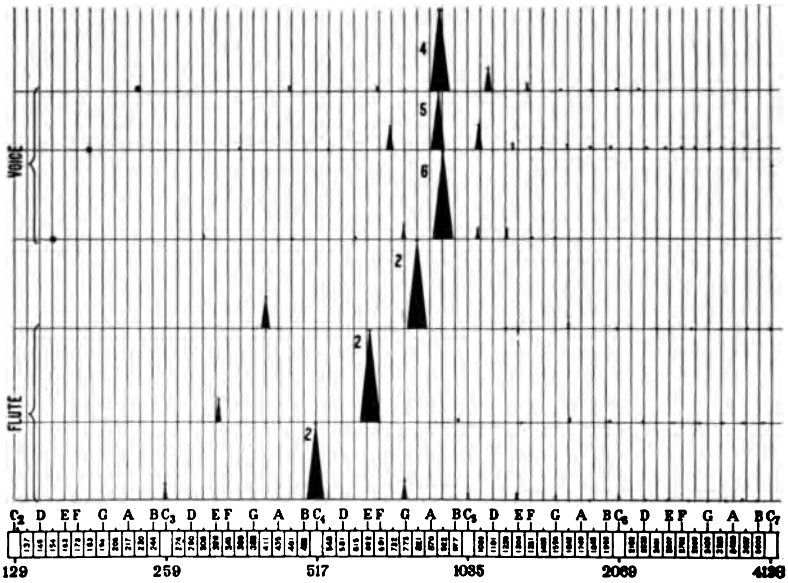


FIG. 185. Analyses of voice and flute tones.

It may be asked what is the effect of raising the pitch of the flute tone a little higher than shown, till its maximum loudness agrees with that of the vowel f*a*ther. When the flute sounds $A_3\sharp = 461$, since the second partial is the loudest, the maximum energy is at $A_4\sharp = 922$, and the tone actually has a resemblance to the vowel. By stopping the breath, somewhat as is done in speaking the word *pa-pa,* the flute imitates the vowel.

"Opera in English"

The characteristics of the several vowels, which were described in the previous lecture, are shown, superposed, in Fig. 186. The essential part of each vowel is a component tone or tones, the pitch of which is within certain limits indicated by the corresponding curve. A characteristic, the

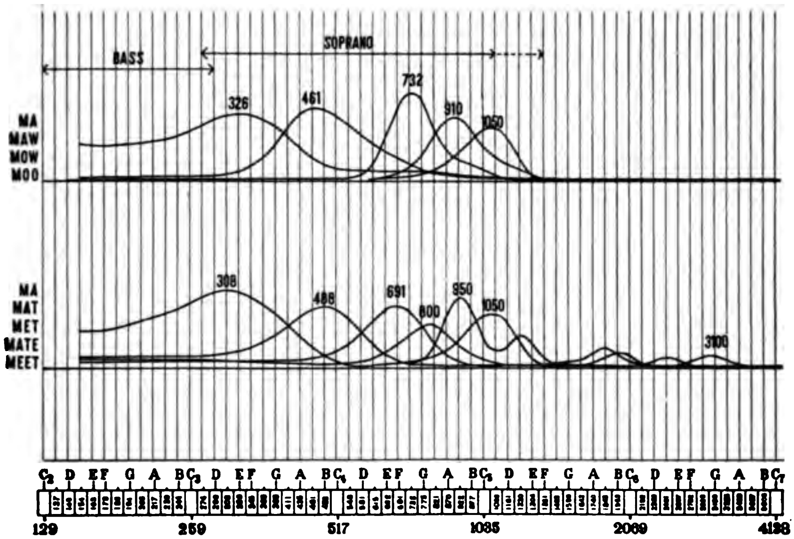


Fig. 186. Superposed curves of vowel characteristics showing relations to voice ranges.

pitch of which comes in the region where the curves for two vowels overlap, would produce an intermediate vowel which might serve for either of those specified, and, although the pronunciation of such a vowel might be different from that used in defining the characteristics, it would be readily interpreted when used in a word. This shaded pronunciation of a vowel would be accepted in singing, where perfect enunciation is not expected. Many singing tones have pitches above the characteristic ranges of certain vowels, such as gl*oo*m and m*ee*t, and these vowels cannot be sung properly at the higher pitches. The vowel f*a*ther has a characteristic higher than any singing tone and can be sung under any circumstances of voice or pitch. Thus there is a scientific reason for the free use of such syllables as *tra-la-la* in vocal exercises. Yodeling is probably the easy flowing of varying vowel tones to fit the melody.

The characteristics of the several vowels are given in musical notation in Fig. 187; the notes correspond to the pitches

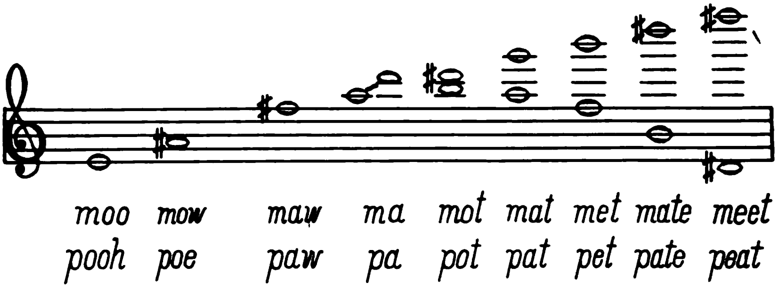


FIG. 187. Characteristics of the vowels in musical notation.

of maximum resonance, as shown by the curves of the previous figure, and each vowel can be most clearly intoned upon the corresponding note. A vowel can also be freely intoned upon any lower note of which the characteristic note is a harmonic, such as notes an octave, a twelfth, or a fifteenth lower (in musical intervals) than the characteristic. A baritone voice can easily intone the vowel gl*oo*m, in falsetto, upon the characteristic pitch of E_3, shown in the figure. The characteristics of all the vowels can be verified by the test of free enunciation.

A consideration of the characteristics of the vowels leads to certain definite conclusions regarding the question, so widely discussed, whether grand opera originally written in a foreign language should be sung in English. No doubt every composer sets words to music with some regard for effective rendition, in doing which he conforms, perhaps unconsciously, to the natural requirements. Suppose that in the original the composer set the vowel r*aw*, having a characteristic pitch of about 732, to the melody note $F_4\sharp$, of the same pitch, the vowel can then be sung and enunciated with ease. If, in the translation, some other vowel, as n*o*, the characteristic pitch of which is 461, falls upon this note its proper enunciation will be difficult, or impossible, since it must be sung at the pitch 732. The vocalist in attempting to sing the vowel will find the result vocally deficient and the effort perhaps physically painful, and will be emphatically of the opinion that translated opera is impracticable. Furthermore, the auditors will hardly understand the English words with the forced and imperfect vowels any better than they understood the foreign language. If the translator arranges the vowels upon the same notes as were used in the original, or upon others equally suitable, the translated opera, so far as this element goes, will be just as satisfactory to both the vocalist and the auditor as was the original. The effectiveness of vocal music is not dependent upon the nationality of its words, but upon the suitability of melody to vowels, a condition which the composer fulfills through his artistic instinct. The translator of an opera must secure this adaptation by his skill; he needs to be not only a linguist and a poet, but also a musician and even somewhat of a physicist, since he must constantly be guided by the facts represented in the curves of vowel characteristics; such a combination of artist and scientist is very rare.

It has been suggested that certain songs and choruses which are especially effective owe this quality to the proper relation of vowel sounds to melody notes, and the Hallelujah Chorus from the "Messiah" has been cited as an instance.

When one considers the authors of the arguments which have been published concerning translated opera, it will be found that some soprano singers are opposed to the translation, while many of those who favor it are baritones. The lines at the top of the diagram, Fig. 186, indicate the ranges in pitch of soprano and bass voices. The pitch of the soprano voice in singing often rises above that of the characteristics of certain vowels, which then become difficult; the bass voice when highest is still below the lowest vowel characteristic, and it is thus able to intone any vowel on any note which it can sing; to one, translated opera seems ineffective, while to the other it causes no difficulty.

Relations of the Art and Science of Music

In the lectures now brought to a close we have very briefly explained the Science of Musical Sounds and have incompletely described some of the methods and results of Sound Analysis. The science of sound is related to at least three phases of human endeavor, the intellectual, the utilitarian, and the æsthetic. In conclusion we will refer to some of these relations without extended comment since an adequate discussion would require several lectures.

The appreciation of knowledge for its own sake is general; and what knowledge should be more valued than that concerning sound, related as it is to many of the necessities as well as to the luxuries of existence? It is true of the science of sound, as well as of all others, that

"The larger grows the sphere of knowledge
The greater becomes its contact with the unknown."

Helmholtz, Koenig, and Rayleigh, by observation, experiment, and theory, have developed this science to magnificent proportions, yet the realm of nature is so vast and varied that some other indefatigable discoverer may be able to push forward into unknown regions, and climbing the height of some discovery, see the inspiring prospect of new, though far distant, truths, and be thrilled with the desire for their possession. The challenge of the unknown and the joy of discovery inspire him to devote himself to further exploration.

While formerly the regard for science was largely confined to the academic world, within the last few years there has arisen a remarkable and widespread appreciation of scientific methods, and now industrial and commercial enterprises are appealing to the scientist for assistance. No sooner is a new scientific fact or process announced than there is an inquiry as to its usefulness. The science of sound will be found ready to satisfy the utilitarian demands which will be made upon it.

The science of sound should be of inestimable benefit in the design and construction of musical instruments, and yet with the exception of the important but small work of Boehm in connection with the flute, science has not been extensively employed in the design of any instrument. This can hardly be due to the impossibility of such application, but rather to the fact that musical instruments have been mechanically developed from the vague ideas of the artist as to the conditions to be fulfilled. When the artist, the artisan, and the scientist shall all work together in unity of purposes and resources, then unsuspected develop-ments and perfections will be realized. These possibilities are becoming manifest in relation to the piano, the organ, and some other instruments.

The artistic and æsthetic musician has been wont to disparage, if not ridicule, the development of mechanical musical instruments, such as the player-piano, yet I believe that, since the establishment of the equally tempered musical scale by Bach, nothing else will have contributed so much to the æsthetic development of musical art. The inventor of the Pianola little dreamed that the mechanical operation of the piano would lead to a thorough scientific study of music and musical instruments, but such has been the result. The simple mechanical player has been developed into elaborate devices for the complete reproduction of artistic performances. The success of these synthetic musical instruments depends upon an analytical knowledge of all the factors of sound and music; that is, the pure science of these subjects must be brought to bear upon the practical problems involved. The mechanical precision of such instruments reacts critically upon the artist-performer and the composer, resulting in greater artistic perfection; the unlimited technical possibilities of the machine is an incentive to the composer to write music with greater freedom.

The marvelous inventions of the telephone and the talking machine could never have been developed without the aid of pure science; a knowledge of the science of electricity and magnetism and of mechanism is not sufficient for their perfection; an increased knowledge of the science of sound is also required.

The utilitarian application of the science of sound is nowhere better illustrated than in the design of auditoriums;

for of what avail is a perfected musical instrument controlled by a master, or of what effect is an oration pronounced with faultless elocution, if the auditors are placed in surroundings which distort and confuse the sound waves so that intelligent perception is impossible?

The artistic world has rather disdainfully held aloof from systematic knowledge and quantitative and formulated information; this is true even of musicians whose art is largely intellectual in its appeal. The student of music is rarely given instruction in those scientific principles of music which are established. Years are spent in slavish practice in the effort to imitate a teacher, and the mental faculties are driven to exhaustion in learning dogmatic rules and facts. Bach said "music is the greatest of all sciences"; while this comparison may not be true at the present time, yet the construction of the equally tempered scale is clearly scientific, and it is no doubt true that the relations of the major and the minor scales, and the nature of chords and their various forms and progressions, as well as many other fundamental principles, can be explained better by science than by precept. Experience indicates that a month devoted to a study of the science of scales and chords and of melody and harmony, will advance the pupil more than a year spent in the study of harmony as ordinarily presented.

Regarding the art of music Helmholtz says: "Music was forced first to select artistically, then to shape for itself, the material on which it works. Painting and sculpture find the fundamental character of their materials, form and color, in nature itself, which they strive to imitate. Poetry finds its material ready formed in the words of language. Architecture has, indeed, also to create its own forms; but they are partly forced upon it by technical and not by purely artistic considerations. Music alone finds an infinitely rich but totally shapeless plastic material in the tones of the human voice and artificial musical instruments, which must be shaped on purely artistic principles, unfettered by any reference to utility as in architecture, or to the imitation of nature as in painting, or to the existing symbolical meaning of sounds as in poetry. There is a greater and more absolute freedom in the use of the material for music than for any other of the arts. But certainly it is more difficult to make a proper use of absolute freedom, than to advance where external irremovable landmarks limit the width of the path which the artist has to traverse. Hence also the cultivation of the tonal material of music has, as we have seen, proceeded much more slowly than the development of the other arts."

In "Music and the Higher Education," Professor Dickinson says: "Strange as it may seem that notes 'jangled, out of tune and harsh,' should give pleasure to any one of average intelligence, yet the abundance of evidence that they do so indicates that the training of the youthful ear to discrimination between the pure and the impure is not to be neglected. . . . The guide to musical appreciation need not deem his effort wasted when he preaches upon the need of preparing the auditory sense to catch the finer shades of tone values. . . . Let the music lover not be content with imperfect intonation, let him learn to detect all the shades of timbre which instruments and voices afford, let him train himself to perceive the multitudinous varieties and contrasts which are due to the relative prominence of overtones . . . and while his ear is invaded by the surge and thunder of the full orchestra, let him try to analyze the thick and luscious current into its elements, . . . turning the dense mass of tone color into a huge spectrum of scintillating hues!"

For the fullest accomplishment of these ends the musician may well appeal to science. Such mathematical and physical studies as we have described prepare the "infinitely rich, plastic material" out of which music is made, and they provide the methods and instruments for its analytical investigation. Nevertheless, that which converts sound into a grand symphony and exalts it above the experiments of the laboratory is something free and unconstrained and which therefore cannot be expressed by a formula. The creations of fancy and musical inspiration cannot be made according to rule, nor can they be made upon command.

Wagner, one of the most inspired musicians that the world has ever known, was offered a great sum of money by the Centennial Commission to compose a Grand March for the opening exercises of the Philadelphia Exposition of 1876. Under the base influence of mere gold which he needed to pay debts, he wrote a very ordinary piece of music, quite unworthy of himself, and of which he was ashamed. In contrast to this, while under the inspiration of the death of a mythical hero in one of his great music dramas, Wagner wrote the Siegfried Death Music, sometimes called the Funeral March, which as performed at Bayreuth is perhaps the greatest and most sublime piece of instrumental music ever heard by man; it is a most profound expression of abstract grief.

In "The Mysticism of Music" the late R. Heber Newton says: "Our modern world is not more distinctively the age of science than it is the age of music; music is the art of the age of knowledge. Music is an emotional symbolism, suggesting that which, as feeling, lies beyond all words and thoughts. 'Where words end, there music begins.' Music can never cease to be emotional, because thought, in proportion as it is deep and earnest, always trembles into feelings. The most holy place in the universe is the soul of man. All sciences lead us up to the threshold of this inner creation, this unseen universe, throw the door ajar and point us within. All arts pass through the open door into the vestibule of this inner temple. Music takes us by the hand, boldly leads us within, and closes the door behind us." (These sentences have been selected from the first twenty-nine pages.)

Music is indeed a mysterious phenomenon; tones, noises, rhythms, time values, mathematical ratios, and even silences, all conveyed to the ear by mere variations in air pressure, are its only means of action, yet with these it awakens the deepest emotions.

Du Maurier, in "Peter Ibbetsen," describes the impressiveness of musical sounds as follows: "The hardened soul melts at the tones of the singer, at the unspeakable pathos of the sounds that cannot lie; . . . one whose heart, so hopelessly impervious to the written word, so helplessly callous to the spoken message, can be reached only by the organized vibrations of a trained larynx, a metal pipe, a reed, a fiddle string — by invisible, impalpable, incomprehensible little air-waves in mathematical combinations, that beat against a tiny drum at the back of one's ear. And these mathematical combinations and the laws that govern them have existed forever, before Moses, before Pan, long before either a larynx or a tympanum had been evolved. They are absolute!"

I would like to quote again from the "Letters of Sidney Lanier," the poet-musician. "'Twas opening night of

Thomas' orchestra, and I could not resist the temptation to go and bathe in the sweet amber seas of this fine music, and so I went, and tugged me through a vast crowd, and after standing some while, found a seat, and the *bâton* tapped and waved, and I plunged into the sea, and lay and floated. Ah! the dear flutes and oboes and horns drifted me hither and thither, and the great violins and small violins swayed me upon waves, and overflowed me with strong lavations, and sprinkled glistening foam in my face, and in among the clarinetti, as among waving water-lilies with flexible stems, I pushed my easy way, and so, even lying in the music-waters, I floated and flowed, my soul utterly bent and prostrate."

And again Lanier writes of an orchestral performance, in which he himself was the principal flutist: "Then came our *pièce de résistance*, the 'Dream of Christmas' overture, by Ferdinand Hiller. Sweet Heaven — how shall I tell the gentle melodies, the gracious surprises, the frosty glitter of star-light, and flashing of icy *spiculæ* and of frozen surfaces, the hearty chanting of peace and good-will to men, the thrilling pathos of virginal thoughts and trembling anticipations and lofty prophecies, the solemn and tender breathings-about of the coming reign of forgiveness and of love, and the final confusion of innumerable angels flying through the heavens and jubilantly choiring together."

A musician must be skilled in the technic of music, he must be trained in musical lore, and above all else he must be an artist; when to these qualifications is added inspiration, the conditions are provided which have given to the world its greatest musicians. But will not the creative musician be a more powerful master if he is also informed in regard to the pure science of the methods and materials of his art? Will he not be able to mix tone colors with greater skill if he understands the nature of the ingredients and the effects which they produce? Does not the interpretative musician need a knowledge of the capacity, possibilities, and limitations of the tonal facilities at his command, as well as a knowledge of the construction of the written music, in order that he may render a composition with consummate effect? While the science of music, because of its incomplete development, has never exerted its full influence on the art, yet it should be appreciated and more generously cultivated for the great assistance which it can be made to yield.

Not only will the creative and interpretative artist be the better able to control the purely mechanical means of operation because of complete knowledge, but the receptive musician will derive greater pleasure from this physical phenomenon if he is also cognizant of its marvelous but systematic complexity. The musically uncultivated and scientifically untrained listener may greatly enjoy music, but this enjoyment is a gratification of the senses. If to this pleasure of sensation is added the intellectual satisfaction of an understanding of the purposes of the composer, the facilities at the command of the interpreter, and the physical effects received by the hearer, then music truly becomes a source of exquisite delight which so pervades and thrills one's being that he is carried away "on the golden tides of music's sea." No other art than music, through any sense, can so transport one's whole consciousness with such exalted and noble emotions.

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Printed in the United States of America.